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## Flashback Teaching Technique Applied to a Block-and-Gap Physics Course


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A new method of teaching physics for general education is described. One principal feature of the method is to use the atom as the central theme of the course. The subject material is divided into three main topics, (1) background material, (2) the external atom, and (3) the nucleus. Organization of the subject material is analyzed with block-and-gap terminology. The other principal feature of the method is to introduce the Bohr model of the hydrogen atom at the beginning of the course and develop only the necessary background material such as Newton's laws of motion, electrostatics, wave motion, etc., as needed. This flashback approach has been found to be very helpful in providing the necessary motivation for the students. Other advantages of the method are the integration of subject material into one completely organized unit and the emphasis on modern physics which makes the course terminal in the desired sense for nonscience students.


### INTRODUCTION

A RECENT study sponsored by the American Association for the Advancement of Science shows that more than half (59 percent) of the four-year colleges in America now have college science courses designed for general education.[1] Among the important trends disclosed by this study is the increased adoption of the case or problem method of instruction in preference to the survey method. Many examples of this trend are described in detail in the volume edited by McGrath.[2]

During the fall semester of 1948–1949 we offered our first version of general education physics in the newly adopted general education curriculum at New Mexico College of A. and M. A. as a typical one-semester 4-hour survey course. The results were quite unsatisfactory to both the students and the instructors. In addition to the inherent lack of unity in subject material, the course was not terminal, in the desired sense of the word, for a general education curriculum. A rather surprising student opinion was that topics which we had labeled "fundamental" did not seem to lead to anything really worth while or conclusive; what the students asked for was less "fundamental" physics and more atomic and nuclear physics, which had been covered very sketchily at the conclusion of the course. A review of this first teaching of the course in the light of this opinion, and also of Rogers' introductory chapter in McGrath's book, led to the question: *What is really the fundamental topic in physics?*

This question doubtless has a wide variety of

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[1] B. B. Watson, Am. J. Phys. **17**, 526 (1949). (This is a brief summary of the work of R. A. Bullington.)

[2] E. J. McGrath, *Science in General Education* (W. C. Brown Company, Dubuque, Iowa).

answers. The most generally accepted answer, however, would probably be *mechanics;* this topic is the starting point of most physics texts and courses because it is used in all other topics of physics. The choice of mechanics as the fundamental topic of physics is fine for physics majors and engineering students, but for the student in the general education curriculum the question needs very critical re-examination.

Rogers[3] has proposed a method of analyzing subject material which is especially useful in the analysis of a general education physics course; in fact, its applicability probably can be extended to a general education course in any of the sciences. The block-and-gap type of course recommended by Rogers for general education is one in which certain related topics are chosen for relatively intensive study while other unrelated topics are omitted completely. In order to provide the necessary unity between the related blocks in such a course it becomes extremely important to point out the interconnections between the blocks, such interconnections being the "life-blood" of the course.

It is here that the flashback technique becomes a logical addition to the block-and-gap scheme since it gives a means of maintaining the continuity which the block-and-gap scheme does not necessarily provide; moreover, it provides a working base to which one may return to organize and integrate the ideas which have been presented. The flashback technique is a method of making transitions from the fundamental block of the course to auxiliary blocks, then back to the fundamental blocks. The term "flashback technique" is in many respects simply a different name for the "project" method used by educationists. The purpose of this paper is to describe our method of treatment of a block-and-gap course using a flashback technique. The course embodying these ideas was first offered at New Mexico College of A. and M. A. during the spring semester of 1949 and, with slight modifications, has been offered during both semesters of the 1949–1950 school year.

Dr. G. W. Gardiner, head of the Physics Department at New Mexico College of A. and M. A. contributed many of the original ideas for the course and has been our most helpful critic during its development. Valuable suggestions also have been offered by Professor H. Bartel Williams and Professor Sanford C. Gladden who have taught sections of the course.

### BASIC PLAN OF COURSE

Our first step in applying the flashback technique to the block-and-gap scheme was to arrive at our version of the fundamental topic of physics. Having chosen this topic, we proceeded to select the subtopics which would form the blocks for our block-and-gap diagram. The naturalness with which these blocks fell into place gave us a great deal of confidence in our choice of a fundamental topic. The third step was to determine the interconnections between the blocks; it was here that the idea of the flashback teaching technique was developed.

The objectives of a one-semester 4-hour course in physics for general education obviously must be kept within modest limits. Our major objectives are to impart an understanding of (a) what physics is about, (b) how physicists investigate and think about natural phenomena, (c) the nature of some of the important problems confronting physics today, and (d) to give the student practice in following the development of a major physical concept from its origin to its logical conclusion. Among the minor aims are the development of a modest scientific vocabulary, the ability to read and evaluate mature popular scientific literature, and the ability to exercise intelligent and reasonably well-informed judgment on scientific matters involving physics.

We now return to the question: What is the fundamental topic in physics? Since physics can be considered as the science of matter and energy, and since the atom is the basic entity of all matter and the ultimate source of all energy, then it follows that the atom can be considered as the fundamental topic of physics. This viewpoint relegates mechanics, electricity, wave motion, etc., to the category of tools used by the physicist in his study of the atom. There is perhaps no other topic in physics which utilizes so many of the basic tools of the physicist as does the atom, hence it provides a strong unifying central theme for a subject matter course. Other choices of a central theme are possible, but it

[3] Eric M. Rogers, Am. J. Phys. 17, 532 (1949). (Also Chapter 1 of reference 2.)

appears doubtful whether they would be as unifying as the atom.

Having chosen the atom as the central theme, the subject material of the course is organized under three main headings, (1) basic tools, including mechanics, electrostatics, wave motion and spectra, (2) the external atom, dealing principally with the Bohr theory, and (3) the nuclear atom, including studies of nuclear processes and nuclear forces on an elementary level.

This basic plan seems to be very nearly the same as that used at Yale University for several years and incorporated in the newly published textbook by Humphreys and Beringer.[4] In fact, this text has been selected for our use during the 1950–51 school year. This plan may well be in use in other colleges, and this paper makes no claims as to its prior originality. The important fact of its independent development is an exceedingly strong confirmation of its soundness. The plan is certainly not limited by the size or type of college in which its use is being considered.

### BLOCK-AND-GAP ANALYSIS

Figure 1 is the block-and-gap diagram for our course. The area of each block is proportional to the approximate amount of class time devoted to that block. The time-area scale has been chosen so that an orthodox one-semester course, covering all the standard topics of physics, would be represented by a single solid rectangle of width equal to "orthodox coverage" and length equal to the "orthodox calendar." This corresponds to the Alpha course described by Rogers,[3] the "orthodox calendar" being an arbitrary average based on teaching experience. The "orthodox coverage" width depends chiefly on the amount of class time available per unit of calendar time and will be only half as wide for a one-semester orthodox course as for a two-semester orthodox course. In our case the area for the orthodox course would be the product of 4 class hours per week times 15 weeks, or 60 class hours; the class actually meets 5 hours weekly for a total of 78–80 hours per semester, the additional hours being devoted to reviews and tests. The total area of the blocks shown, not counting connecting links, is equal to 60 class hours, since the orthodox course and the block-and-gap course are assumed to utilize equal amounts of class time. The block-and-gap diagram has been normalized to the orthodox course.

No attempt has been made to fix a mathematical ratio of length to width of each block; when the area of a block has been determined, the ratio of coverage within the block compared to orthodox coverage is estimated simply by judgment from teaching experience. These ratios are somewhat arbitrary and tend to be different when determined by individual instructors. On the basis of the block shapes chosen in this case the amount of subject material covered is only about 40 percent of the total subject material covered in the orthodox course; this decrease is compensated for by a corresponding increase in depth of coverage. Motion, Newton's laws, and work and energy receive about twice the coverage afforded in the equivalent one-semester orthodox course, while the atom receives at least 4 times the one-semester orthodox coverage. These factors may appear large, but they are quite reasonable to anyone who has ever

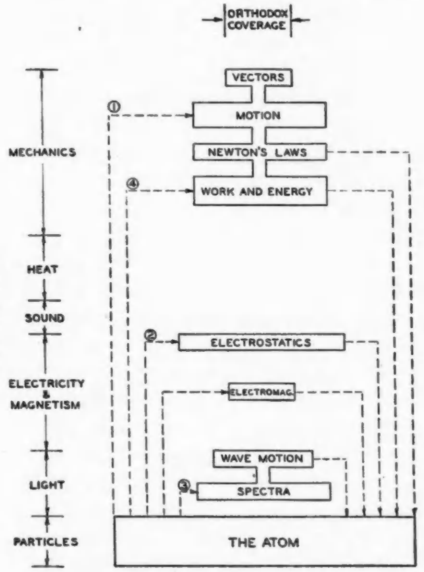


FIG. 1. Block-and-gap diagram.

[4] R. F. Humphreys and R. Beringer, *First Principles of Atomic Physics* (Harper and Brothers, New York, 1950).

attempted the almost impossible task of covering all the major topics of physics in a one-semester course.

The gaps are equally as important as the blocks since the presence of the gaps permits the expanded coverage given to most of the blocks. Mechanics is actually covered quite adequately, which shows its necessity as a tool in understanding the atom. Heat and sound are not covered at all. As far as sound is concerned, it is merely a special case of wave motion, which can easily be omitted. Heat is not so easily dispensed with, however, especially kinetic theory. This omission has caused us considerable concern but so far it has appeared to be more justified than the omission of any of the other blocks. One of the cardinal principles of Rogers' block-and-gap scheme is that there must be gaps; if a new block is added to a course already full, then its equivalent must be subtracted somewhere else. The two blocks in electricity and magnetism appear rather meager, but here again for the study of the atom it is not at all necessary to study current electricity and electrical machinery. The treatment of electromagnetism is very brief and consists essentially in establishing the concept of motor force for the understanding of the $e/m$ experiment and mass spectroscopy. The gaps in light are very large, but as in the case of electricity and magnetism, the omissions are scarcely noticed. The atom block accounts for nearly half of the course time. This rather large share of time has been dictated partly by the lack of a suitable text, a large part of the material having been given to the students through the time-consuming process of lecture notes. The adoption of Humphreys and Beringer's text is expected to reduce this time substantially and to permit the incorporation of new blocks into the course such as one on kinetic theory. This is not a contradiction of Rogers' principle stated above, but is an increase of the total course content through more efficient teaching methods. A subdivision of the atom into the outer atom and the nucleus, and the further division of these blocks into such blocks as the hydrogen spectrum, periodic table, x-rays, $e/m$, etc., could be made but has been avoided to keep the block-and-gap diagram as simple as possible.

Two types of interconnection are shown on the diagram, the difference being partly for ease in pictorial representation. The interconnecting links are used chiefly between adjacent blocks which come under one of the traditional major subject headings such as mechanics; for blocks under different major subject headings the interconnections are shown as dashed lines. Only the most important interconnections of this type have been shown. The principal connections between the atom block and the classical topics are labeled 1, 2, 3, 4, and will be discussed in greater detail in the following section. The preponderance of links between the atom block and the other blocks shows the strong unifying influence of the atom as the central theme of the course.

Although it is desirable to have a connecting link between every block and the atom block such connections are not always essential. An example of this is the vectors block. It serves to give a better understanding of motion and Newton's laws, and hence is used only indirectly as far as the atom is concerned. Two small blocks which have been eliminated because of an insufficiently strong link to the central theme of the atom are momentum and projectile motion. There are certainly good reasons for the inclusion of these topics, but for our purpose, momentum is covered fairly well by Newton's third law and projectile motion is unnecessary. Careful and repeated study of the interconnections is of great value to the instructor in his daily preparation. It is easy for the instructor to overlook links which help give the student a unified picture of the subject material. This unified picture is very valuable in promoting learning by reasoning rather than by rote. The interplay of theory and experiment is often revealed through interconnections, a good example being that between the hydrogen spectrum and the Bohr theory.

## FLASHBACK PRESENTATION TECHNIQUE

Motion picture producers often use a flashback technique as an effective method for arousing and maintaining the interest of the spectator in a plot which might, otherwise, be exceedingly dull if presented in chronological order. Thus, the introductory scene of a murder mystery is usually the murder itself, the major part of the film being devoted to filling out the background with the

necessary motives and suspects, and finally concluding with the apprehension of the criminal. In our adaptation of this technique on a different level and time scale, the atom is used as the introduction to the course. Since most students nowadays already have a speaking acquaintance with the atom, they find themselves on somewhat familiar ground. Numerous questions immediately arise, and the need for obtaining background information to answer these questions becomes apparent to the students at the beginning of the course, the time when it is most needed. Approximately the first half of the course is devoted to filling in background material, the second half then being the development of the simple atomic picture presented at the introduction.

Several examples will perhaps assist in explaining the actual treatment of the flashback technique. Part of the introductory lecture is used to point out the relationship of physics to the field of scientific knowledge in general and includes a brief historical sketch of the development of physics. The importance of the atom as a unifying concept of physics and as the choice of the central theme for the course then is emphasized. The lecture concludes with the film *Atomic Energy*.[5] The second class period is designed more as a discussion than as a cut and dried lecture, although the discussion must be guided to the desired goal. The discussion centers on some of the details of the Bohr model of the hydrogen atom, which has been illustrated in the film of the first lecture. The usefulness of powers of 10 immediately becomes apparent and this shorthand notation is developed. Following this, the discussion is led into consideration of the motion of the electron about the nucleus and its analogy in the motion of the earth about the sun. The questions which now arise are "Why does the electron revolve about the nucleus?" and its parallel "Why does the earth revolve about the sun?" It is pointed out that Newton asked himself the same question about the earth and the sun and that his successful effort to answer this question gave us his laws of motion and gravitation. The students are now alert to the fact that Newton's laws also may provide the answer to our question about the hydrogen atom, hence the motivation for studying Newton's laws has been established. However, in stating these laws briefly the instructor uses such terms as "force," "acceleration" and "velocity" which must be defined before the laws can be completely understood. Thus, the flashback to the study of both motion and Newton's laws *via* interconnection (1) in the block-and-gap diagram is accomplished. A period of about two and a half weeks is devoted to these two blocks plus the vectors block, during which time repeated applications are made to the hydrogen atom as well as to the more familiar examples ordinarily used in mechanics. Having filled in this part of the background material, we return to the atom.

The students now discover that whereas the gravitational attraction is sufficient to provide the proper centripetal force on the earth, it does not provide a sufficiently large centripetal force on the electron. This establishes the motivation for a digression into electrostatics *via* interconnection (2) on the block-and-gap diagram. The coverage of electrostatics has been greater than absolutely necessary because it is a topic in which the students have a natural curiosity, and it becomes, therefore, an important interest factor. This is an example of a compromise between the double necessity of making a block fairly complete within itself, while at the same time restricting the material covered to the basic background material necessary to continue with the study of the atom. Once again we return to the atom.

The next step is to demonstrate that a collection of atoms (gas) emits a line spectrum when properly excited. This poses the question as to whether a study of spectra will give clues regarding the behavior of the atom. In this manner the students are brought to the point where the need for a study of the nature of spectra and wave motion becomes apparent; interconnection (3) on the block-and-gap diagram is thus exploited. This interconnection is a very intimate one and leads inevitably to interconnection (4), the study of work, energy, energy levels and quanta.

These four principal flashbacks complete the background necessary for a detailed study of the atom. From this point on the course proceeds to

[5] *Atomic Energy*, Encyclopedia Britannica Films Inc., Chicago, Illinois.

its conclusion in a more orthodox manner. The course becomes self-motivating; in fact, it is usually difficult to restrain some of the students in their eagerness to get to the heart of the nuclear physics discussion. Guided discussions are promoted in preference to the straight lecture approach except during certain demonstration periods when a fixed lecture plan becomes a necessity.

The value of the flashback technique as applied to a block-and-gap course is that the method promotes a maximum exploitation of the interconnections between blocks. The choice of the atom as the central theme of the course permits natural and logical transitions to nearly every major branch of physics; this is very important to students having a limited background in physics because to them the usual transitions from one branch of physics to another are apt to seem abrupt and illogical. This scheme, in addition to maintaining the interest of the students and providing the motivation for a deeper penetration into physics, also has other advantages. It can be used to demonstrate the interdependence of physical principles from the time of Galileo to the present; it shows the interlocking of the various branches of physics and intimates how each of these branches is built upon certain fundamental concepts common to all of physics—in other words, the unity of physics; and above all, it can be used to show how physicists work and think. As the foundation block of the course, the atom has the attribute of giving rise to a logical and progressive sequence of questions culminating in the physicist's atomic concept of the physical universe.

## INSTRUCTIONAL TECHNIQUES FOR THE COURSE

Prior to evaluation of the course it may be worthwhile to give a brief description of some of the instructional techniques used. Classes meet one hour per day, five days per week. This anticipates a nominal division into two demonstration or laboratory days and three discussion days.

Planning, preparation, and presentation of demonstration lectures are carried through with the utmost care, since they are expected to be a primary medium of instruction as well as the means of creating interest and providing additional motivation. This means that a considerable amount of time must be spent on their preparation but the results justify the efforts. It has been the policy in this department to rotate the demonstrations among the various instructors and have the instructor assigned to a given lecture present it to all sections. There are advantages to such an arrangement, but student reaction to the change of instructors has not been too favorable and consideration is being given to the plan of having each instructor give the demonstrations to his sections and to no others.

The laboratory possibly is one of the weakest parts of the course. From the first there has been a question, still unresolved, as to what part it should play. Without entering into the pros and cons of the discussion it appears fairly obvious that the orthodox type of physics laboratory experiment is entirely unsuited for this course. Therefore, the questions arise, "Should any laboratory work be given? If so, what form should it take?" In discussing the organization of the course it appeared that the two most convincing reasons for having laboratory experiments were as follows: first, since physics is basically an experimental science any attempt to teach the course without giving some idea of the problems besetting experimental procedures, the techniques involved in overcoming or circumventing these problems, and the tie-in between experimental results and theoretical deductions, would result in a one-sided and distorted idea of how physicists think; and second, a laboratory properly organized and conducted would be an additional source of interest and motivation to the students. The unique nature of the block-and-gap organization limits the experiments which can be fitted into the course. Moreover, it obviates the need of lengthy weekly laboratory periods. Experiments are scheduled during the regular class periods, and continue over two or more periods when necessary.

A group of five experiments is in current use in the course. The first one, carried out as a demonstration experiment, involves the experimental determination of the equation $s=\frac{1}{2}at^2$ by means of the inclined plane. Emphasis in this experiment is twofold: first is the demonstration of the details of an experimental setup and the measurement of physical quantities; second is

the correlation of the data to give a useful law of motion. This experiment is followed in a few days by the determination of $g$ through direct measurement of $s$ and $t$, applying the result of the inclined plane experiment. This is also a demonstration experiment with student participation. The third and fourth experiments on vectors and forces and the electroscope are individual student-work experiments. The last experiment, the most popular of the group, is concerned with line spectra, and is performed by small student groups. In spite of its popularity this experiment presents many difficult problems to the instructor, and has been given in three different versions with a fourth one planned for future use. Because of the unsettled state of affairs this experiment will not be described further. There is a definite need for new ideas in connection with the laboratory part of the course.

The discussion part of the course will vary considerably with the institution and with the individual instructor, and though the plan used at New Mexico College of A. and M. A. is merely an application of proved ideas, it still may be of some interest. In general, the various sections of the course are coordinated very closely, all following the same outline which incorporates the main ideas of the course. This outline is worked out in a series of conferences by members of the staff teaching the course. This is a time-consuming procedure but seems to be necessary where the various sections are closely coordinated. It also has an important advantage in that a free discussion among the teaching members of the course brings out new ideas and tends to eliminate poor ones.

Testing methods, naturally, are of utmost importance to the students. The four or five one-hour tests are preceded by very thorough review periods which provide an excellent opportunity for re-emphasizing the interconnections between topics. These hour tests are made up primarily of carefully chosen objective (multiple choice) type questions, the major emphasis being on subject material. The questions are prepared carefully with a view toward emphasizing basic concepts and stimulating student thinking. Thoughtful and painstaking preparation is an absolute necessity in order that these questions become an integrating link of the course. It is very important that the questions be read and proofed carefully by someone other than the person preparing them because students in this course, not having a broad background in physics, often derive implications which were never intended in the questions. Another important part of the testing procedure is the giving of from fifteen to twenty 10-minute quizzes during the semester. These are usually, though not always, announced ahead of time. Students favor announced quizzes over unannounced ones, and since the former produce more satisfactory results it appears that they are better suited to the purpose of inducing study by the student. No definite form is followed in these quizzes but a large share involve discussion questions. Such questions can be used to cover rather broad phases of the course which are difficult to cover with objective-type questions. The time-limiting nature of the 10-minute quizzes is obviously beneficial to both students and instructor where discussion questions are involved. The two-hour final examination is made up entirely of objective type questions. All quizzes and tests are written with as little reference to previous quizzes or tests as possible except for idea material. This procedure, although time-consuming, nevertheless is helpful to the instructor in providing the opportunity and incentive for careful study and analysis of all course material.

Assigned problems also play a part in the plan of presentation. There is nothing new in the use of problems, but in this case they are tailored to the course with more than the usual care. From 75 to 100 problems are assigned each semester, and the list is completely revised for each succeeding semester.

Every attempt is made to keep the course flexible. Questions and free discussions by the students are encouraged if they pertain in the slightest degree to the subject material being studied or even if they are considered to be contributory to a general and useful knowledge of physics. A constant lookout is maintained for newspaper and magazine articles which might be of interest and form the basis of useful and pertinent discussion. Such current items often add a dramatic element which can be turned to the dual purpose of creating interest and at the

same time providing a subject for critical analysis from the viewpoint of the physicist.

## EVALUATION OF THE COURSE

When properly chosen, the objectives of any course also should be the standards for evaluating the achievements and success of the course. We shall restrict this evaluation to the opinions of members of our staff who have either taught the course or who have followed its progress closely, and to a summary of the main results obtained from questionnaires filled out by students at the end of the semester.

Although the course involves a great deal of preparation and conference time, this is compensated for by a refreshing manner of approach usually not found in orthodox courses taught at this level. The student, completely unfettered by any previous knowledge of physics, often will ask questions which tax the ingenuity of the instructor to answer satisfactorily in terms understandable to the student. Although the course is a required one, it is not intended to be made so difficult that students will be driven from it or will anticipate it with dread. On the other hand, it is not intended to be easy. Our hope is for a happy medium which above all will stimulate the students to think and reason. It is our opinion that the intellectual level of the course is high, certainly far above that of a one-semester survey physics course, and in certain respects above that of the orthodox two-semester liberal arts and engineering physics courses. This is doubtless made possible through the depth of penetration involved in the block-and-gap approach. The ability to visualize an atom, which can never actually be seen, is certainly a good test of one phase of intelligence. The emphasis is placed on physical concepts and physical principles rather than on a collection of unrelated and probably unassimilated factual data. For example, the implications of Newton's laws are stressed rather than the various techniques of applying, let us say, the $F=ma$ equation. Every effort is made to induce the students to think, to analyze physical processes, and to make an interpretation of theoretical deductions drawn from experimental facts. Our opinion in regard to the major objectives stated earlier is that objective (a), what physics is about, is achieved very well within the subject matter limits of the course; objective (b), how physicists investigate and think, is only moderately successful, partly because of difficulties in devising suitable testing methods; objective (c), the important problems of physics today, is achieved moderately well, again within the subject matter limits; while objective (d), the development of a major physical concept, appears to have been accomplished very successfully. This element of subject matter limits is inherent in a block-and-gap course and emphasizes once again the advantage of choosing the atom as the backbone of the course, since it can be extended to cover nearly all of physics. Of the minor aims the only one which can be judged with any degree of success is the one of acquiring a scientific vocabulary. This perhaps is achieved more successfully than in an orthodox course because of the early introduction and consequent repeated usage of most of the vocabulary.

A fairly reliable indication of student interest is the attendance record. An analysis shows an average of approximately 2 unexcused absences per student per semester under a cut system which permits 6 unexcused absences.

Opinions from 55 students (90 percent), as summarized from the questionnaires completed in June 1950, tend to confirm our own opinions. The choice of ratings available to the students on the intellectual level of the course were low, medium, and high, with about 65% indicating medium, most of the remainder indicating high. There was only one low rating. In rating the course as very easy, easy, difficult, or very difficult, the last two categories were checked most frequently; a few students rated the course as easy, but no one classified it as very easy. A very few students rated the course boring, more rated it mildly interesting, and a majority rated it very interesting. In commenting on which one of ten listed abilities had been most greatly improved by the course the five abilities checked most frequently were "ability to understand the physical world," "···to visualize physical phenomena," "···to read and understand science in the news," "···acquire a physics vocabulary," and "···to think critically," in that order. The response to a question as to whether more or less coverage should be given to the various topics

was that about 4 out of 5 were in favor of more material on atomic and nuclear physics. Since we believe that our coverage of these topics is quite adequate we interpret this comment as an indication of the popularity of the atom as the central theme of the course. It is believed that so long as the course draws such comments as the above from the students, a good beginning toward its success has been achieved.

### FUTURE POSSIBILITIES

Many variations of the plan as presented are possible. For example, the progress of the physicist's concept of the atom from pre-electron times to the present could be treated; the flashback technique could be used in such a plan with equal success, but the order of events would have to be somewhat altered. Although the block-and-gap plan can be used for a two-semester as well as a one-semester course, it is rather doubtful whether the flashback technique could be applied successfully for more than a few weeks. It is a technique which definitely must not be over-extended. The advisability of using a text incorporating the flashback technique (if such a text were available), has been considered, and it is our opinion that such a text would probably do more harm than good. The choice of the atom as the central theme of the course also appears to offer promise for the full year physics course for nonscience students. It would be a simple and logical procedure to go from kinetic energy to a study of heat and kinetic theory; or from wave motion to sound; or from the motion of the electron in the atom to electrodynamics; and so on. In fact, such a procedure might be applied to good advantage in engineering physics courses.

One limitation of using the atom as the main topic of a terminal course for nonscience students is that it is difficult to carry the picture of the atom very far beyond the simple Bohr mechanical model. Our treatment of this particular problem has been to substitute the energy level model of the atom for the mechanical model *after* the mechanical model has served its purpose.

Changes have been and will continue to be made in our version of this course, but the essence of the course is its flexibility, and changes are an indication of its aliveness. Criticisms of this course as outlined above and comments concerning similar types of courses will be appreciated.

## Manual of Advanced Undergraduate Laboratory Experiments

The American Association of Physics Teachers is undertaking to produce a manual of advanced undergraduate laboratory experiments in physics as a memorial to the late Lloyd William Taylor of Oberlin College. So far as possible, this is to be a cooperative undertaking, and the active participation of all those interested in this field of teaching is urgently requested. It is planned that this book will comprise all fields of physics, to enable an instructor to plan and set up experimental work on this level in any field or fields of his choice. The undersigned committee undertook this project in February 1950. A more complete report of its work to date, and its plans for the future, will appear in a later issue of this *Journal*.

Contributions may include new experiments, improvements upon standard experiments, recommendations concerning experiments published in past issues of this or other journals (including the contributors' own experience with them), lists of standard experiments which should be included in the bibliography, and areas in which new experiments are needed.

These contributions should be sent to the editor, JOSEPH D. ELDER, Harvard University Press, 44 Francis Avenue, Cambridge 38, Massachusetts.

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# On Newton's Law of Attractions

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A careful analysis of the demonstration of his fundamental law of attractions, set forth by Newton in Proposition LXIX of Book I of the *Principia*, seems to show that the objective of the proof is not to be attained without first assuming the very points which are there sought to be established. As soon as we proceed on the basis of such assumptions, the subject puts on a different appearance, which is deeply helpful in appreciating afresh the connection between a gravitational and inertial measure of masses. There is brief secondary treatment of the so-called tautologous character of Newton's definition of material density and of a field concept implied in newtonian description of accelerative forces.

## I

NEWTON'S law of attractions is contained in Proposition LXIX, of Book I of his *Principia*, which reads:

> *In a system of several bodies A, B, C, D, &c., if any one of these bodies, as A, attract all the rest, B, C, D, &c., with accelerative forces that are reciprocally as the squares of the distances from the attracting body; and another body, as B, attracts also the rest, A, C, D, &c., with forces that are reciprocally as the squares of the distances from the attracting body; the absolute forces of the attracting bodies A and B will be to each other as these very bodies A and B to which these forces belong.*
>
> For the accelerative attractions of all the bodies *B, C, D,* towards *A*, are by the supposition equal to each other at equal distances; and, in like manner, the accelerative attractions of all the bodies towards *B* are also equal to each other at equal distances. But the absolute attractive force of the body *A* is to the absolute attractive force of the body *B* as the accelerative attractions of all the bodies towards *A* to the accelerative attractions of all the bodies towards *B* at equal distances; and so is also the accelerative attraction of the body *B* towards *A* to the accelerative attraction of the body *A* towards *B*. But the accelerative attraction of the body *B* towards *A* is to the accelerative attraction of the body *A* towards *B* as the mass of the body *A* to the mass of the body *B*; because the motive forces, which (by the 2d, 7th, and 8th definitions) are as the accelerative forces and the bodies attracted conjointly, are here equal to one another by the third law. Therefore the absolute attractive force of the body *A* is to the absolute attractive force of the body *B* as the mass of the body *A* to the mass of the body *B*.
>
> Q. E. D.

This Proposition is applied in Proposition VII of Book III to arrive at the proportionality of gravitational force of bodies to the quantities of matter which the bodies contain. It is thus of extremely fundamental importance.

## II

It seems to me that the Proposition has not been demonstrated, and this demonstration of it rather needs the assumption of what is sought to be established. I submit the considerations here below.

From Rule III of "Rules of Reasoning in Philosophy," Book III: ". . . Not that I affirm gravity to be essential to bodies: by their *vis insita* I mean nothing but their *vis inertiae*. This is immutable . . ."

If so universal a force as gravity may not be deemed as immutable for bodies, far less can that be asserted of another (excepting of course *vis insita* which is of the nature of *vis inertiae*). There is nothing indeed in the notion itself of a force that can be impressed—by Definition IV in Book I, an impressed force is an action which changes or tends to change inertial states of bodies—which can impose limitation on its quality or on the proportion in which it is generated, or in other ways characterize it specifically. Limitations of this nature can be only a matter of an initial assumption about a particular force or of an observational ascertainment of such features. And, in fact, in the Scholium that immediately follows the Proposition LXIX, Newton expressly draws attention to the position that up to that stage the question of the physical nature, qualities, or species of the forces considered is an entirely open and indeterminate one.

The mutability of a force may be, of course, manifestation of several concomitant or separate

factors, like variation with distance from the seat of force, physical species of the force, e.g., among given metallic pieces of equal mass identically treated for magnetization and existing in such free spaces that the variations in the accelerative quantities of their forces with distance would there depend on the distance only; i.e., the law of force depends on the distance only,[1] the one of iron will be more energetic in its action piece for piece, and in this way various particular proportions will obtain among the pieces in respect of their absolute forces. If any exception is taken to my mention of a magnetic force in this regard, I may defend myself that a loadstone is taken by Newton as one cause of such force, in the brief illustration that follows the Definition of Centripetal Force. Moreover, there is reference, under Definition of the motive quantity of a centripetal force, to "anything else" being admitted as a cause of force "but which does not yet appear"; and about which, therefore, we can assert nothing regarding the character of mutability of its force (except that we stipulate here that as regards the mutability of its accelerative force with distance it would follow a definite law depending on the distance only; i.e., the law of force depends on the distance only). And in this Proposition LXIX this is the situation; as already observed, in the Scholium that succeeds the Proposition the physical species or qualities themselves of the forces are expressly mentioned to be indefinite and undetermined.

Now when mass $A$ attracts mass $B$, the former is drawn itself towards mass $B$ with force equal, by the third law, to the motive force exerted on $B$. And similarly in respect of the action of $B$ on $A$. Now because the motive force on $B$ is proportional to its mass (by the Definition of the motive quantity of force),[2] the force with which $A$ is drawn towards $B$ by the reaction of $B$ is therefore indeed proportional to the mass of $B$. (It must be noted carefully that we are here referring only to the reaction of $B$.) But we can as yet assert nothing about the original action that proceeds from $B$, i.e., the absolute force of mass $B$, or for that purpose from $A$, being proportional to the mass of $B$ or of $A$. For the third law merely asserts the equality of original action and the reaction to it, and in no wise lays down the quantities or proportions of the original actions. Remembering from the preceding discussion that a force (other than *vis inertiae*, which is indeed not a force in the general sense) is not any immutable quality of bodies, and that whether universal or special it is of the nature of a mutable action, the absolute forces issuing out of $B$ and $A$ may be the one to the other, for aught we know without an assumption or an empirical ascertainment, as the values of some functions $f(M)$ and $f(m)$ of their respective masses. For it might well be that as the aggregation of mass is greater, the absolute force thereof gets stimulated or depressed corresponding to fluctuations in the value of some function $f$ of the mass. We arrive at the position that, in order to carry through a demonstration of the content of the Proposition, we have to make the assumption (and a very sweeping one at that) that any and every component part of the masses $A$ and $B$ has an unvarying absolute force of its own without reference to, or influence of, other parts. This is precisely tantamount to what is being sought to be proved.

For let us imagine two bodies $A$ and $B$, with masses $M$ and $m$, and $R$ the distance separating them. We designate the absolute forces tending to them as functions $f(M)$ and $f(m)$. Then the accelerative forces would be $kf(M)/R^2$ and $kf(m)/R^2$; $k$ is a constant of proportionality as to distance-effect. Then the motive forces on $B$ and $A$ that would result from the action of these accelerative forces are $kf(M)\times m/R^2$ and $kf(m)\times M/R^2$. In virtue of the third law, $B$ would itself as well experience the force $kf(m)\times M/R^2$ towards $A$, and similarly $A$ would itself as well experience the force $kf(M)\times m/R^2$ towards $B$. Therefore, the total force by which $A$ and $B$ would be drawn towards each other is, of course, the same, viz., $[kf(M)\times m/R^2]+[kf(m)\times M/R^2]$. The total force being the same, the accelerations $A$ and $B$ would manifest towards each other,[3] respectively, $A$ towards $B$ and $B$ towards $A$, would be $[kf(M)\times m+kf(m)\times M]/R^2M$ and

[1] See Appendix I.
[2] Besides, of course, being proportional to the operating accelerative force.

[3] Acceleration in an absolute sense is meant, and not relative to bodies.

$[kf(M)\times m+kf(m)\times M]/R^2m$; i.e.,

$$\frac{\text{the acceleration } B \text{ would experience towards } A}{\text{the acceleration } A \text{ would experience towards } B}$$

is as the mass of $A$/mass of $B$. Similarly, if $A$ were to be replaced by body $C$ of the same characteristics (as indeed the same characteristics are assigned to $C$ in Cor. 1 to this Proposition) but of mass $M'$,

$$\frac{\text{the acceleration } B \text{ would experience towards } C}{\text{the acceleration } C \text{ would experience towards } B}$$

is as the mass of $C$/mass of $B$. But these consequences or ratios of factors pertaining *inter se* to pairs of bodies (or more of them, as the case may be) can tell us nothing about the ratio of the absolute force of $A$ to the absolute force of $B$, etc., these consequences being merely an index of the fact that equal, but in contrary directions, force is acting as between $A$ and $B$, $C$ and $B$, and so on, whatever the absolute forces themselves be and however they may vary. It is

$$\frac{\text{acceleration that } B \text{ would experience towards } A}{\text{acceleration that } B \text{ would experience towards } C}$$

that would represent the ratio of the absolute force of $A$ to the absolute force of $C$. We develop the position with regard to the absolute forces of $A$ and $B$. [Also see Appendix II for further information.]

Let us take the body $E$, of mass $n$, and imagine it similar in all respects to $A$ and $B$ (as indeed set out in Cor. 1 of this Proposition). I shall call this mass $n$ by the name of "test-particle." We conceive the test-particle as placed, alternately, at distance $r$ from $A$ and at same distance $r$ from $B$. The distance being the same,

$$\frac{\text{absolute force of } A}{\text{absolute force of } B}$$

would be equal to

$$\frac{\text{acceleration test-particle would experience towards } A}{\text{acceleration test-particle would experience towards } B}.$$

Now

$$\frac{\text{acceleration test-particle would experience towards } A}{\text{acceleration test-particle would experience towards } B}$$

$$=\frac{\dfrac{\dfrac{kf(M)\times n}{R^2}+\dfrac{kf(n)\times M}{R^2}}{n}}{\dfrac{\dfrac{kf(m)\times n}{R^2}+\dfrac{kf(n)\times m}{R^2}}{n}}=\frac{kM}{km}\left\{\frac{\dfrac{f(M)}{M}+\dfrac{f(n)}{n}}{\dfrac{f(m)}{m}+\dfrac{f(n)}{n}}\right\}. \qquad (1)$$

The right-hand expression in Eq. (1) can only be set equal to $M/m$ either by assumption that $f(M):f(m)=M:m$ or any actual observational values (or inferences from them) which allow us to equate it to $M/m$. This is as it should be, for the notion itself of force furnishes no clue and leaves matters undetermined. [Also see Appendix II as already mentioned.]

The strength of the Proposition has lain, in my opinion, in the correspondence of what is rather assumed in it to empirical facts relating in particular to gravity. It is significant that in the application of this Proposition to gravity, after the initial reference to it, Newton also seeks to establish the proportionality of absolute force of gravity being as the mass by means of express analogies with the empirically widely ascertained knowledge of magnetic attractions (Cor. 1, Prop. VII, Book III). And no less significant is the circumstance that the demonstrative character of the remarkable Propositions and Theorems LXX–LXXVI of Book I remains wholly unaf-

fected by our reasonings. For there the requirement is only that the centripetal forces tending to each of the "points" of a spherical body are equal among those very points (and of course diminish as we recede from a point with the inverse square law). It is not at all required, for the demonstration of these Propositions and Theorems, that the centripetal force tending to a point of a spherical body is to be equal to the centripetal force tending to a similar point of another spherical body (of course, at equal distances with obedience to the inverse square law), which is of a greater or smaller mass or density than the aforementioned one. That is a matter for assumption or for an initial empirical test.

## III

It is in the subtle confusion represented in the demonstration of this Proposition and its application to gravity that there lay, in the opinion of the writer, the root of the obscurities that hung over the correct relationship of the so-called "gravitational" and "inertial" masses which found a detailed clarification in the General Theory of Relativity, taken as a whole. (By "gravitational" mass is meant here the mass' coefficient of efficacy in generating a gravitational field of force. This is the sense in which the word is to be understood hereafter throughout in this article.)

Newton drew a fundamental distinction between *vis insita*, which is thought of as an immutable *vis inertiae*, and gravity, which has no such exalted status and is indeed not affirmed even "to be essential to bodies." But the "demonstration" of this very fundamental Proposition is seen to demand as a basis the assumption of the strict proportionality of absolute force of bodies concerned to the quantity of matter which they contain. And the proportion of absolute force of gravity of masses rests in theory on this Proposition, as set out in Prop. VII of Book III of *Principia*. We thus see that the unvarying ratio of "inertial" and "gravitational" masses of a body resides in a deep theoretical necessity. If we do not enter thoroughly into the structure of this fundamental Proposition, and consider its content (erroneously) as a rather mere consequence of a particular application of the third law, we are wont to regard, as has been the case always, the unaltering correspondence of "inertial" and "gravitational" measures of masses as only a conspicuous coincidence which need not be invested with anything more profound. And such has been the case until the General Theory of Relativity threw new lights on the subject, as a whole.

## IV

This Proposition refers to absolute forces of the attracting bodies $A$ and $B$ being as these very bodies $A$ and $B$, i.e., as their masses or quantities of matter (by Definition I in Book I, "body" also stands for "mass" or quantity of matter), and to accelerative forces as being as the squares of the distances (or varying more generally with the distances, as mooted in Cor. 2) from the attracting body. I should not, therefore, feel it amiss, on the occasion of the consideration of this Proposition, to express also, but in just a few words, some part of my reflections on Newton's Definition of the "quantity of matter," and what I may call "Newton and the Concept of the Field."

*1.*—The definition of the quantity of matter is given as Definition I, Book I, viz., "The quantity of matter is the measure of the same, arising from its density and bulk conjunctly." Almost every commentator on this definition has experienced a more or less uncomfortable feeling over what he collectively regard as its tautologous character; e.g., as recently as 1935 the noted philosopher-scholar Professor A. Wolf draws attention to this feature on page 154 of his well-known *History of Science, Technology and Philosophy in the XVI and XVII Centuries*.[4]

In my opinion, Newton is concerned in this Definition not so much with a formal definition as in giving expression to what is basic in his reasoning, viz., the absolute distinction of "matter" from "space." He is here seeking, I believe, to direct attention to the degree of "materiality" of a region (see also Newton's explanation of "place of a body" that is given hereafter in Sec. IV, 2), a degree of "materiality" he wishes to be deemed more primitive than any estimations of it resting on the physical interactions in, or associated with,

[4] A. Wolf, *History of Science, Technology and Philosophy in the XVI and XVII Centuries* (The Macmillan Company, New York, 1939).

a mass in any region. It is interesting to find that the late Professor W. de Sitter had drawn attention to such fundamental status of material density in Newton's reasoning (p. 28 and p. 29 of his fine little book *Kosmos*).[5] We find this fundamental character of density emphasized again in Cor. 4 to Prop. VI of Book III, where the most basic part of Newton's mechanics, *vis inertiae*, is being referred very deliberately to the material density.

*2. Newton and the Concept of the Field.*—Accelerative quantity of a centripetal force is characterized in the explanation of it that follows its definition in Definition VII as: ". . . I refer . . . the accelerative force to the place of the body, as a certain power or energy diffused from the center to all places around to move the bodies that are in them. . . ." And "place of a body" is characterized, in item III of the Scholium to the Definitions, as: "Place is a part of space which a body takes up . . . I say, a part of space; not the situation, nor the external surface of the body. For the places of equal solids are always equal; but their superficies, by reason of their dissimilar figures are often unequal . . . the place of the whole is internal, and in the whole body."

From these two statements, taken together, it appears to me not entirely unclear that in Newton's mind the *general* conception of a centripetal force is being considered also, in some way or other, as something coextensive and concurrent with the surrounding space, and not as a wholly mechanical action, whether one of "at a distance" or of "contact transmission" in a medium. Let us now recall to mind Clerk Maxwell's "abstract" definition of the electric field, viz., "The Electric Field is the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena. It may be occupied by air or other bodies, or it may be a so-called vacuum, from which we have withdrawn every substance which we can act upon with the means at our disposal."[6]

At least a semblance between the statements of Newton and Maxwell's is not totally indistinct. Indeed, without some such abstract notions about the properties of portions of space encompassing an attraction-center, the Cors. 3–8 of Prop. IV of Book I and Props. I, II, III of Book III seem hardly unequivocal. For how can we genuinely know that the centripetal force concerned would ever "rise to the occasion" in proportion to the "greatness of mass" of the bodies concerned so that the notified proportion of its variations with distance is the real distance-effect and not some fortuitous coincidence arising from the peculiar proportions of the quantities of matter of the few masses concerned (which masses are undetermined up to that stage)[7] and their distances from the attraction center, in virtue of which the true law, if any, connecting the force and distance is rather masked? It must be carefully borne in mind, to appreciate a validity of the suggestions here being advanced, that at this stage Newton is not starting from the fact of universal gravity, of which we know from abundance of experience that it would ever "rise to the occasion" in proportion to the "greatness of the masses" with which it deals, but is trying rather to work, step by step, towards it. And with regard to gravity also he speaks of its force, in one of his penetrating reflections so often encountered in his writings, as *not* operating as "mechanical causes used to do" (General Scholium at the end of Book III).

In conclusion, it is a pleasure to express my thanks to Mr. K. B. Abdullah, Librarian of the Panjab University Library in Lahore, Pakistan, for allowing me the privilege of full access to the library.

## APPENDIX I

The establishment of the Proposition is generalized in Cor. 2. I need scarcely point out that mention there of "law of force" depending on the distance only has no reference to the qualities or proportions of the absolute forces, *which proportions indeed are the object of enquiry.* This mention of the "law of force" has reference only to the law of variation of the accelerative quantity of the forces with distance, that the law of variation of the accelerative quantity of the forces depends on the distance as such and not upon any con-

[5] W. de Sitter, *Kosmos* (Harvard University Press, Cambridge, 1932).

[6] James Clerk Maxwell, *Treatise on Electricity and Magnetism* (Oxford University Press, London, 1904), third edition, Sec. 44.

[7] Rather, the conclusions being arrived at are utilized to compare this class of masses.

tingent physical factor co-extensive with the distance or otherwise in attendance.

This is the more readily obvious if you will see that the reference is to accelerative attractions which, by the 7th Definition and Newton's illustration of it, depend on the place of attracted bodies, so that the law of force concerning accelerative forces has to do only with places and changes of places. This is to say, that the law of force regarding accelerative forces is an expression of the changes in accelerative forces with the changes in distances of the attracted bodies from the attracting center, or, in other words, any law of force here depends on the distance only.

**APPENDIX II**

We have worked at the discussion directly in line with Newton's procedure in this Proposition that the absolute forces are as accelerative forces at equal distances. And the accelerative forces are proportional, by the 7th Definition in Book I, to the velocities which they generate in a given time. (In our discussion I have used, instead of "the velocity generated in a given time," the more familiar word "acceleration.")

If the situation were that the attracted test-particle had no attractive virtue of its own, then for bodies $A$ (mass $M$) and $B$ (mass $m$) their absolute forces indeed quite simply would be as

$$\frac{\text{acceleration test-particle would experience towards } A}{\text{acceleration test-particle would experience towards } B}.$$

But in the more general, and important, case of the text the "test-particle" has also an attractive virtue of its own. We must not, therefore, neglect to indicate a certain variation in the procedure followed in the text. This is connected with the following consideration. Because the test-particle, in attracting mass $A$ or mass $B$, is itself drawn, by the third law, towards $A$ or $B$, the ratio

$$\frac{\text{acceleration test-particle would experience towards } A}{\text{acceleration test-particle would experience towards } B}$$

may not be the right measure of the ratio of the absolute forces of $A$ and $B$.[8] (It is to be distinctly understood that up to this stage we have not *assigned* how the absolute force is to vary with the mass.)

The acceleration test-particle would experience towards $A$ or $B$ is made up of the accelerative attraction proceeding from $A$ (or $B$) and acceleration arising from the reaction on test-particle to its motive attraction of $A$ (or $B$). And since test-particle's motive attraction of $A$ (or $B$) depends, besides the mass of $A$ or $B$, on accelerative attraction proceeding from test-particle, it is the ratio of acceleration a test-particle would experience towards $A$ to the acceleration *another* test-particle would experience towards $B$, where the mass $n$ of the first test-particle is to the mass $n'$ of the second test-particle as the mass of $A$ to the mass of $B$, that would represent the ratio of the absolute force of $A$ to the absolute force of $B$. For the acceleration a test-particle is to experience would arise from the interaction between the mass of the attracting body and the test-particle's own attracting mass, the effect of the attracting virtue of test-particle operating on itself through the third law in the manner already explained above. And because it is only such "compound" accelerations that we have to deal with, it is not very difficult to realize that, for comparing the absolute forces of bodies $A$ (mass $M$) and $B$ (mass $m$) by this means, the sums, respectively, of pairs of interacting masses, $M$ and $n$, and $m$ and $n'$, must bear the same ratio to each other as the masses $M$ and $m$ bear themselves, which, of course, means that the mass $n$ is to the mass $n'$ as $M$ is to $m$. It is not so well put in words, but a reference to the elementary mathematical formulation that follows would provide a full clarification. In the meantime I may say here, while passing on to these elementary equations, that the reader should notice further (which is quite easy) that unless an attractive virtue of masses is at this stage *posited* as proportional to the masses, all bodies will *not* fall towards a given mass with equal accelerations as observed by Galileo and confirmed by the delicate test of Baron Eotvos.[9] We can now readily see, by the

[8] This does not interfere with the conclusion itself in the text.

[9] If a mass $p$ attract all the masses $p_1$, $p_2$, $p_3$, etc., with accelerative forces which are as the inverse squares of the distances from $p$, and likewise is the case with $p_1$, $p_2$, $p_3$, etc., then the accelerations $p_1$, $p_2$, $p_3$, etc., would experience

aid of our line of study, the intimate connection that subsists between three factors, viz., the measure of attractive virtue of masses, the accelerations that bodies of different mass would experience from the same point towards an attracting body, and the "inertial" mass.

Let us denote, as in the text, the accelerative attractions of $A$ and $B$ at distance $r$ from each, as $kf(M)/r^2$ and $kf(m)/r^2$, where $f(M)$ and $f(m)$ are representative of the absolute forces and $k$ is a constant of proportionality as to distance-effect. The test-particle regarding $A$ is of mass $n$ and that regarding $B$ is of mass $n'(n'=mn/M)$, so that $(M+n)/(m+n')=M/m$. Then following previous procedure in form,

$$\frac{\text{absolute force of } A}{\text{absolute force of } B}=\frac{\text{acceleration test-particle of mass } n \text{ would experience towards } A}{\text{acceleration test-particle of mass } n' \text{ would experience towards } B}$$

$$=\frac{\dfrac{knf(M)+kMf(n)}{nr^2}}{\dfrac{kn'f(m)+kmf(n')}{n'r^2}}=\frac{n'}{n}\times\frac{nf(M)+Mf(n)}{\dfrac{m}{M}nf(m)+mf\left(\dfrac{m}{M}n\right)}=\frac{n'}{n}\times\frac{nf(M)+Mf(n)}{\dfrac{mn}{M}f(m)+\dfrac{mf(m)\times f(n)}{f(M)}}=\frac{f(M)}{f(m)},$$

since

$$\frac{n'}{n}=\frac{m}{M}.$$

It is now again, as before, a matter for *stipulation* or inference from experimental facts to decide on the equality of $f(M)/f(m)$ to $M/m$ (or to some other relationship).

It is manifest that the considerations contained in this paper are entirely of a physical nature and do not take any directions whatever from mathematical means employed. For this reason, the concrete mathematical formulations adopted above are selected for their straightforwardness only, e.g., the functions are here assumed to be ones where $f(mn)=f(m)\times f(n)$ holds. And therefore, we need not enter into the question of a formal extension to unrestricted functional condition, since it does not in any way deepen the essential insight already gained.

I am indebted to my extremely able friend, Dr. Mohammad Raziuddin Sidiqi, Director of the Research Institute in Peshawar (Pakistan), for penetrating suggestions I received from him.

towards $p$ at distance $r$ would be $[kp_1\times f(p)+kp\times f(p_1)]/p_1r^2$, $[kp_2\times f(p)+kp\times f(p_2)]/p_2r^2$, etc., which are not equal unless $f(p)=p$, $p$ denoting mass, in general.

*When we approach any new branch of knowledge we nearly always find that the things which interest the specialists are not the obvious things which we should expect. Thus we might expect the Law to be chiefly concerned with such questions as the penalty for murder, whereas in fact the lawyer will hardly give it a thought, and will occupy his whole time with trying to make a precise definition of what constitutes a "contract." In the same way we might expect a book on light to discuss why the grass looks green, and gold yellow, but such matters are hardly mentioned. Instead, a chief part of the discussion is devoted to phenomena almost unknown to ordinary experience, the fact that light does not always go in straight lines—a thing which most go through life without consciously observing. This gives an unreal and sometimes pedantic look to the studies of the scientist, because he always seems to forget about the broad facts and to be concerned with petty details.* —C. G. DARWIN, *The New Conceptions of Matter*, 1931.

# On Order Parameters*

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The problem of describing the state of order in a binary alloy is considered. A set of order parameters is defined in the following way: $p_i$ is the probability that if a given site is occupied by a $B$ atom then a site which is its $i$th neighbor is occupied by an $A$ atom. The relationships of this set to the parameters used by other authors are discussed.

THE problem of describing the state of order in an alloy is one which has engaged the attention of physicists for some years[1–3] since the determination of the proper terms for the description of order is an essential step in understanding the nature of ordering in the alloy. Once suitable order parameters are defined one can hope, on the one hand, to relate the observed order-dependent properties (electrical resistance, x-ray scattering, specific heat, etc.), to these order parameters and, on the other hand, to calculate the order parameters as functions of the temperature by the methods of statistical mechanics. Although one finds considerable discussion of order parameters in the literature, the author feels it worthwhile to discuss the subject again for two reasons. First, one finds a variety of definitions of order parameters; the discussion is often carried out in terms of variables which provide an essentially incomplete description of order. Furthermore, there is no adequate discussion of the interrelations of the parameters used by different authors. Second, no clear distinction seems to have been made in most of the literature between the problem of defining order parameters and that of calculating such parameters. It is perhaps a trivial observation that the problems are separable, but it is apparently not so widely realized, as one might expect, that one can define an adequate set of order parameters without going into the complexities of the statistical mechanics of cooperative phenomena.

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[1] F. C. Nix and W. Shockley, Revs. Modern Phys. **10**, 1 (1938).

[2] G. H. Wannier, Revs. Modern Phys. **17**, 50 (1945).

[3] R. H. Fowler and E. A. Guggenheim, *Statistical Thermodynamics* (Cambridge University Press, Teddington, England, 1949), Chap. 13.

In this paper we shall give a discussion of the nature of order in alloys and, following this, define a consistent and complete set of order parameters. We shall then analyze the order parameters used by other authors and show their relationships to the set we define and to each other.

## DEFINITION OF ORDER PARAMETERS

Let us begin by discussing qualitatively the nature of order in an alloy. The state of the alloy is completely determined by assigning the occupation of every lattice site in the crystal. A knowledge of the state in such detail is never possible, of course, nor is it necessary for understanding any of the properties of the alloy. What we do require is a knowledge of how the atoms of the constituents, $A$ and $B$ in a binary alloy, are arranged, on the average, relative to one another. (The term "on the average" will be made precise below.) In particular, it is sufficient for essentially all purposes to know the probability of finding any pair of lattice sites occupied by, let us say, a $B$ atom and an $A$ atom.

If there were no order whatsoever in the crystal, then the occupation probabilities of the two sites would be independent, and therefore the joint occupation probability of the pair of sites would be merely the product of the individual probabilities, which are determined only by the composition of the alloy. In general, however, there will be a correlation between the occupation of different sites: thus, the occupation of a particular site by a $B$ atom will modify the probability of finding an $A$ atom on the other site of the pair. (The $B$ atom will, for example, tend to attract $A$ atoms as its immediate neighbors.) The joint occupation probability is then no longer the product of the individual prob-

abilities as these are no longer independent. Furthermore, we must expect that the joint occupation probability of a pair of sites will depend on the distance between these sites.

Before proceeding, we shall specialize the type of lattice that we consider. We assume a binary alloy whose $N$ atoms are of two types: $m_A N$ of type $A$ and $m_B N$ of type $B$ where, of course,

$$m_A + m_B = 1. \quad (1)$$

For definiteness we take $m_A \geqq m_B$. We shall require that, *in the state of perfect order:* (a) all nearest neighbors of $B$ atoms are $A$ atoms, and (b) of the nearest neighbors of $A$ atoms, a fraction $m_B/m_A$ are $B$ atoms and the remainder are $A$ atoms. Two simple well-known alloys satisfying these conditions are $\beta$-brass (CuZn) and $Cu_3Au$. In the first, $m_{Cu} = m_{Zn} = \frac{1}{2}$; the lattice is body-centered cubic. When the lattice is completely ordered all Cu atoms are on one simple cubic lattice, say the cube corners, and all Zn atoms are on the other lattice, the cube centers. In the completely disordered state all lattice points have equal probability ($\frac{1}{2}$) of being occupied by either Cu or Zn atoms. In the case of the $Cu_3Au$ structure we have a face-centered cubic lattice, $m_{Cu} = \frac{3}{4}$, $m_{Au} = \frac{1}{4}$. In the perfectly ordered state the Au atoms occupy one of the four simple cubic lattices, say the cube corners, and the Cu atoms occupy the other three lattices, the face centers. In the totally disordered state all lattice points are occupied by Cu or Au atoms with relative probabilities 3:1.

We may point out that not all alloys exhibiting order satisfy conditions (a) and (b). For example, the alloy CuAu in the ordered state consists of alternate planes of Cu and Au atoms.

To return to our discussion: we can always write, for the probability of finding a pair of sites which are $i$th neighbors[4] occupied by a $B$ and an $A$ atom, $m_B p[B|A(i)]$; i.e., the joint occupation probability is the probability, $m_B$, of finding the first site occupied by a $B$ atom times the probability $p[B|A(i)]$ that if the first site is occupied by a $B$ atom then the second site is occupied by an $A$ atom. Since the word probability is used in two slightly different senses in the previous sentence, let us give an operational definition of what we mean. The probability of finding the first site occupied by a $B$ atom would be determined by examining each site of the crystal, counting the number of sites occupied by $B$ atoms, and dividing by the total number of sites. This clearly gives us $m_B$. To find $p[B|A(i)]$ we would examine every $B$ atom in the crystal, determine the fraction of its $i$th neighbors which are $A$ atoms, add the fractions so obtained for all $B$ atoms, and divide by the total number of $B$ atoms. From these operational definitions one can clearly see that $p[B|A(i)]$ is a correlation or, more properly, conditional probability. In general, the conditional probabilities $p[B|A(i)]$ do not have the value $m_A$ corresponding to complete randomness, but vary in a systematic way with $i$, which measures the distance between the sites of the pair.[5]

Since the conditional probabilities $p[B|A(i)]$ will be basic to the discussion that follows, it is convenient to introduce a simple notation for them. Let us define

$$p_i \equiv p[B|A(i)]. \quad (2)$$

We may repeat the definition: $p_i$ is the probability that *if* a given site is occupied by a $B$ atom *then* a site which is its $i$th neighbor is occupied by an $A$ atom. A knowledge of the set of variables $p_i (i = 1, 2, 3, \ldots)$ gives us an adequate description of the state of order in the alloy crystal.[6]

Having defined our basic order parameters $p_i$ as $p[B|A(i)]$, the question immediately arises as to whether the corresponding conditional probabilities $p[B|B(i)]$, $p[A|A(i)]$, and $p[A|B(i)]$ are determined by $p_i$. It is readily seen that they

[4] The various shells of neighbors of a lattice site $(h, k, l)$ are defined as follows: one determines the distances from site $(h, k, l)$ to all other sites in the crystal and enumerates these distances as a monotonically increasing sequence. Then the sites whose distance from $(h, k, l)$ is the $i$th term in this sequence constitute the $i$th shell of neighbors of $(h, k, l)$.

[5] The notion of a complete set of order parameters was apparently introduced by Zernike in an important and often neglected paper on "The Propagation of Order," F. Zernike, Physica **7**, 565 (1940). More recent discussions and calculations using the matrix method are found in papers by J. Ashkin and W. E. Lamb, Jr., Phys. Rev. **64**, 159 (1943), and B. Kaufman and L. Onsager, Phys. Rev. **76**, 1244 (1949).

[6] The variables $p_i$ do not determine all correlations within the crystal. One might, for example, want to know the probability of finding an $A$ atom at a given distance from a pair of neighboring sites occupied by $B$ atoms. These other correlation probabilities do not seem to be needed for understanding any of the measurable properties of the crystal although they have a certain intrinsic interest. See the paper of Ashkin and Lamb, reference 5.

are. First, since

$$p[B|A(i)]+p[B|B(i)]=1, \qquad (3)$$

we have

$$p[B|B(i)]=1-p_i. \qquad (4)$$

Second, since

$$m_A p[A|A(i)]+m_B p[B|A(i)]=m_A, \qquad (5)$$

we have

$$p[A|A(i)]=1-p_i m_B/m_A. \qquad (6)$$

Equation (5) follows from the requirement that the probability $m_A$ of finding an $A$ atom on a site (2) which is an $i$th neighbor of some site (1) be equal to the probability $m_A$ of finding an $A$ atom on (1) times $p[A|A(i)]$ plus the probability $m_B$ of finding a $B$ atom on (1) times $p[B|A(i)]$.

Finally, from Eq. (6) and an obvious modification of Eq. (3),

$$p[A|B(i)]=p_i m_B/m_A. \qquad (7)$$

We may now discuss the behavior of the order parameter $p_i$ as a function of $i$ in different temperature ranges. First, at infinite temperatures, i.e., temperatures high enough so that all Boltzmann factors can be taken as unity, the occupation of the lattice sites is random. Hence, all $p_i$ are equal to $m_A$. When the temperature is reduced somewhat we find a tendency to local order. In other words, the odd shells around a $B$ atom, for example, tend to be occupied by $A$ atoms while the even shells are preferentially occupied by $A$ atoms. These correlations do not extend to an unlimited distance, however, but are of finite range, the range increasing as the temperature is reduced. (As the temperature is reduced, the reduction in energy obtained by increasing the order tends to become more important than the increase in entropy obtained by increasing disorder.) For sites beyond this range, i.e., for $i$ greater than some value, the occupation probabilities are again essentially random, $p_i=m_A$.

This situation is depicted in Fig. 1 showing $p_i$ as a function of $i$ in this temperature region. The graph has not been drawn as a smooth curve since only the discrete points for integral values of $i$ have any significance. The behavior of $p_i$ for odd $i$, $p_i^{(o)}$ and even $i$, $p_i^{(e)}$ have been indicated separately with dotted lines. We may note that, in this temperature region,

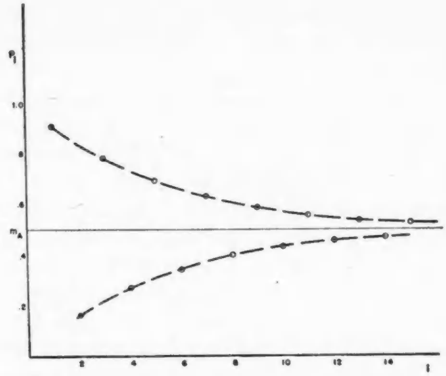

FIG. 1. Schematic graph of $p_i$ as function of $i$ in the absence of long range order ($T>T_c$). ($m_A=\frac{1}{2}$).

$$p^{(o)}=p^{(e)}=m_A, \qquad (8a)$$

where

$$p^{(o)}=\lim_{i\to\infty} p_i^{(o)}; \quad p^{(e)}=\lim_{i\to\infty} p_i^{(e)}. \qquad (8b)$$

Now all alloys of the kind we are considering have a transition temperature $T_c$ marked by anomalous behavior of many properties, the best known of which is the specific heat. The essential feature of the transition temperature is that for $T<T_c$, the nature of the order in the crystal is essentially changed. There is now a new feature present; long range order. In the terms we have chosen to describe order, long range order means that even as $i$ approaches infinity, the conditional probability $p_i$ does not approach its random value; there is a pattern of order maintained

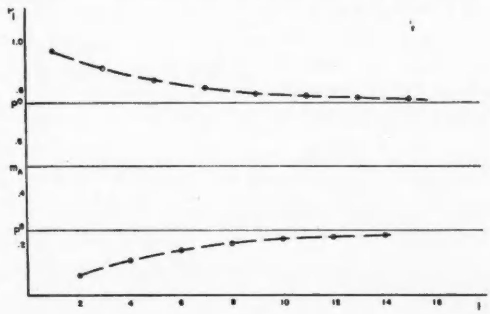

FIG. 2. Schematic graph of $p_i$ as a function of $i$ when long range order is present ($T<T_c$). ($m_A=\frac{1}{2}$, $p^{(o)}=0.75$, $p^{(e)}=0.25$).

throughout the crystal as a whole. This is in contrast to the behavior above $T_c$ which was shown in Fig. 1.

To understand the nature of long range order in more detail let us consider the limits of $p_i$ for odd and even $i$, $p^{(o)}$, and $p^{(e)}$. These two limits are no longer equal to one another for $T<T_c$. Instead, a plot of $p_i$ against $i$ has the form indicated in Fig. 2; for large $i$ there is a regular alternation in the value of $p_i$ as one goes from odd to even shells. As the temperature is lowered still further the difference between $p^{(o)}$ and $p^{(e)}$ increases until, at $T=0$, when there is perfect order, $p_i^{(o)}=p^{(o)}=1$; $p_i^{(e)}=p^{(e)}=0$. In that case the conditions (a) and (b) above are exactly fulfilled.

We may point out that the discussion of the variation of order parameters with temperature, given above, is based on both theoretical and experimental evidence,[7] the most direct being the recent x-ray diffraction studies of Cowley.[8]

## RELATIONSHIPS AMONG ORDER PARAMETERS

We have now defined a set of order parameters $p_i$ which give an adequate description of the state of the alloy. Our next task is to discuss the relationships between the $p_i$ and the parameters used by other authors.

*1. Bragg-Williams Long Range Order Parameter.* Historically the first parameter to be used was the long range order parameter of Bragg and Williams.[9] It is natural that this parameter, whose exact definition is given below, should have been introduced first since, as we have seen, the existence of long range order is perhaps the most striking feature in the behavior of these alloys. The long range order parameter is still a very useful one although the statistical theory of Bragg and Williams, which used *only* this parameter to describe order, is now seen to be only a rather rough first approximation.[1,2]

We may define the Bragg-Williams parameter $S$ in the following way. Let us call the lattice sites which are occupied by $A$ (or $B$) atoms when the lattice is perfectly ordered $\alpha$- (or $\beta$-) sites. For example, in the alloy $Cu_3Au$ the $\alpha$-sites are the face centers, the $\beta$-sites are the cube corners. Now let $r_\alpha$ be the fraction of the $A$ atoms to be found on $\alpha$-sites and let $r_\beta$ be the fraction of $B$ atoms to be found on $\beta$-sites in a particular state of the alloy. Then $S$ is defined by

$$S=(r_\alpha-m_A)/(1-m_A);\quad r_\alpha=m_A+m_BS. \tag{9}$$

Thus, $S$ is the actual value of $r_\alpha$ minus its random value divided by the difference between the value of $r_\alpha$ for perfect order (one) and its random value. We notice that $S=1$ for perfect order and $S=0$ for a completely random occupation of the sites.

Since the fraction of $A$ atoms on $B$ sites can be expressed either as $m_A(1-r_\alpha)$ or as $m_B(1-r_\beta)$ according to whether we consider the $m_A$ atoms of type $A$ or the $m_B$ sites of type $\beta$, we can readily show that

$$S=(r_\alpha-m_B)/(1-m_B);\quad r_\beta=m_B+m_AS, \tag{10}$$

an equation of the form of Eq. (9).

Now to relate $S$ to our parameters $p_i$ we must calculate, in terms of $S$, the probability of finding a pair of sites occupied by $A$ and $B$ atoms. In general, $S$, which measures the type of occupation of $\alpha$- and $\beta$-sites, should depend on all $p_i$. Since, however, for large $i$ the probability of finding an $A$-$B$ pair of $i$th neighbors depends only on whether $i$ is odd or even (i.e., on the type of site in question) we want to express $S$ in terms of the limiting values $p^{(o)}$ or $p^{(e)}$. It must be pointed out that an assumption is made here: namely, we are assuming that $p_i^{(o)}=p^{(o)}$ and $p_i^{(e)}=p^{(e)}$ for all $i$, thus neglecting the deviations of these quantities from their limiting values. This assumption should not be bad well below the transition temperature where the order is not far from perfect.

Granting this assumption, we have

$$m_Bp^{(e)}=m_Br_\beta(1-r_\beta)+m_A(1-r_\alpha)r_\alpha, \tag{11}$$

$$m_Bp^{(o)}=m_Br_\beta r_\alpha+m_A(1-r_\alpha)\times[m_B/m_A(1-r_\beta)+(1-m_B/m_A)r_\alpha]. \tag{12}$$

These expressions are readily understood: thus in Eq. (11) we have equated the probability of finding an $A$-$B$ pair of even order neighbors to the sum of two terms. The first term is the probability ($m_B$) that the first site be a $\beta$-site times the probability ($r_\beta$), that it be occupied by

[7] See references 1 and 5. Also J. M. Cowley, Phys. Rev. **77**, 669 (1950).

[8] J. M. Cowley, J. Appl. Phys. **21**, 24 (1950). Also B. Strijk and C. H. MacGillavry, Physica **11**, 369 (1946); **12**, 129 (1946).

[9] W. L. Bragg and E. J. Williams, Proc. Roy. Soc. (London) **A 145**, 699 (1934). Also see reference 1.

a $B$ atom times the probability $(1-r_\beta)$, that the $i$th neighbor site, which is also a $\beta$-site for even $i$, be occupied by an $A$ atom. The second term of Eq. (11) is similar in structure and arises from the $\alpha$ sites. Equation (12) has a more complicated second term since the odd neighbors of $\alpha$-sites are not all $\alpha$-sites. Substituting from Eqs. (9) and (10) into Eqs. (11) and (12) we find that

$$p^{(e)}=m_A(1-S^2);\; S^2=1-p^{(e)}/m_A; \tag{13}$$

$$p^{(o)}=m_A+m_BS^2;\; S^2=(m_A/m_B)(p^{(o)}/m_A-1). \tag{14}$$

We may notice as a check that when $p^{(e)}$ and $p^{(o)}$ are random (i.e., equal to $m_A$), then $S=0$; when $p^{(e)}=0$, and $p^{(o)}=1$, we find $S=1$ corresponding to perfect order.

We also obtain a relationship between $p^{(o)}$ and $p^{(e)}$,

$$p^{(o)}=1-p^{(e)}m_B/m_A. \tag{15}$$

*2. Bethe Short Range Order Parameter.* As mentioned above, the statistical theory of order developed by Bragg and Williams using only the long range order parameter was found to be inadequate. A step forward was made by Bethe[10] whose theory introduced one more order parameter, which described the order of nearest neighbors. To see the nature of Bethe's parameter, let us introduce a quantity $q$ equal to the fraction of nearest neighbor pairs in the lattice consisting of unlike atoms. Then we define the Bethe parameter $\sigma_1$ by

$$\sigma_1=[q-q(\text{random})]/[q(\max)-q(\text{random})], \tag{16}$$

where $q$ (random) and $q$ (max) are, respectively, the values of $q$ for complete disorder and perfect order. Notice that $\sigma_1=1$ for perfect order and $\sigma_1=0$ for complete disorder.

To determine $\sigma_1$ in terms of $p_1$, which is clearly the only one of the set $p_i$ on which it can depend, we notice that the fractions of nearest neighbor pairs are given by the following relations:

*B-B* pairs: $m_B(1-p_1)$,
*A-A* pairs: $m_A(1-p_1m_B/m_A)$,
*A-B* pairs: $m_Ap_1m_B/m_A$,
*B-A* pairs: $m_Bp_1$,

where we have used Eqs. (2)–(7).

[10] H. A. Bethe, Proc. Roy. Soc. (London) **A 150**, 552 (1935). See also reference 1.

Hence $q$, the fraction of pairs of unlike nearest neighbors is given by

$$q=2m_Bp_1, \tag{17}$$

and therefore

$$\sigma_1=(2m_Bp_1-2m_Bm_A)/(2m_B\cdot 1-2m_Bm_A)$$
$$=(m_A/m_B)(p_1/m_A-1). \tag{18}$$

Now one can generalize Bethe's definition to obtain a set of order parameters analogous in form to $\sigma_1$ but equivalent in scope to our set $p_i$. Let us define $q_i^{(o)}$ as the fraction of $i$th neighbor pairs consisting of unlike atoms (for odd $i$) and $q_i^{(e)}$ as the fraction of $i$th neighbor pairs consisting of like atoms (for even $i$). Then

$$q_i^{(o)}=2m_Bp_i^{(o)};\quad q_i^{(e)}=1-2m_Bp_i^{(e)}. \tag{19}$$

If we define $\sigma_i$ as

$$\sigma_i=[q_i-q_i(\text{random})]/[q_i(\max)-q_i(\text{random})], \tag{20}$$

then we obtain

$$\sigma_i^{(o)}=(m_A/m_B)(p_i^{(o)}/m_A-1), \tag{21}$$

and

$$\sigma_i^{(e)}=1-p_i^{(e)}/m_A. \tag{22}$$

The $\sigma_i$ are all equal to one for perfect order and to zero for complete randomness.

On comparing Eqs. (21) and (22) with Eqs. (13) and (14), we notice that

$$S^2=\lim_{i\to\infty}\sigma_i, \tag{23}$$

showing that the long range order parameter $S^2$ of Bragg and Williams is, so to speak, in the same family as the $\sigma_i$.

The parameters $\sigma_i$ have been used by Wilchinsky[11] in discussing the results of his x-ray diffraction studies of order and disorder.

*3. Other Parameters.* (a) Cowley[7,8] has used the parameters $\alpha_i$ defined by

$$\alpha_i=1-p_i/m_A \tag{24}$$

in his x-ray diffraction work and has shown that they are Fourier coefficients in the expansion of the diffracted intensity in terms of the reciprocal lattice coordinates.

(b) Kirkwood[12] has used a parameter measuring the order of nearest neighbors equal to

[11] Z. W. Wilchinsky, J. Appl. Phys. **15**, 806 (1944).
[12] J. G. Kirkwood, J. Chem. Phys. **6**, 70 (1938). See also reference 1.

our conditional probability $p[A|A(1)]$ i.e., to $1-p_1 m_B/m_A$. (See Eq. (6).)

(c) In an earlier paper with L. Tisza[13], the author has shown that in certain problems of statistical mechanics it is useful to consider the correlation coefficient $g_{\lambda\mu}$ defined by

$$g_{\lambda\mu}=\langle f_\lambda f_\mu\rangle_{\text{Av}}/[\langle f_\lambda^2\rangle_{\text{Av}}\langle f_\mu^2\rangle_{\text{Av}}]^{\frac{1}{2}}. \quad (25)$$

For present purposes we may take $f_\lambda$ as any function defined on the lattice site $\lambda$ and depending only on its occupation. It is simple to evaluate the averages in Eq. (25) in terms of the $p_i$. If $f_A$ and $f_B$ are the values of $f_\lambda$ when site $\lambda$ is occupied by an $A$ or a $B$ atom then we obtain, using Eqs. (2)–(7),

$$\begin{aligned}\langle f_\lambda f_\mu\rangle_{\text{Av}} &= m_A f_A[(1-m_B/m_A p_i)f_A+m_B/m_A p_i f_B] \\ &\quad +m_B f_B[p_i f_A+(1-p_i)f_B] \\ &=(m_A-m_B p_i)f_A^2+2m_B p_i f_A f_B \\ &\quad +m_B(1-p_i)f_B^2. \quad (26)\end{aligned}$$

$$\langle f_\lambda^2\rangle_{\text{Av}}=\langle f_\mu^2\rangle_{\text{Av}}=m_A f_A^2+m_B f_B^2. \quad (27)$$

Now if we choose $f_\lambda$ so that $f_A=0$, $f_B=1$, i.e., if $f_\lambda$ is the characteristic function of the set of lattice sites occupied by $B$ atoms, then

$$\langle f_\lambda f_\mu\rangle_{\text{Av}}=m_B(1-p_i), \quad (28)$$

$$\langle f_\lambda^2\rangle_{\text{Av}}=m_B. \quad (29)$$

Hence we obtain for $g_i$,

$$g_i=1-p_i, \quad (30)$$

where $g_i$ is now the correlation coefficient for finding two $B$ atoms which are $i$th neighbors. Comparing with Eq. (4), we see that, as expected,

$$g_i=p[B|B(i)]. \quad (31)$$

(d) Frenkel[14] employs the correlation coefficient in another manner. He defines local order parameters by a definition like that of Eq. (9) applied to a small region or cell of the alloy crystal and then uses the correlation coefficients of the order parameters in different cells to describe the order. This definition does not seem a convenient one and so we shall discuss it no further.

## CONCLUDING REMARKS

We have attempted to give a unified discussion of the various order parameters that have been used in the literature, relating them all to a standard set of parameters defined in a simple way. It is hoped that this paper will serve a useful purpose by clarifying the nature of the various order parameters and separating the problem of their definition from the much more difficult statistical mechanical problem of calculating these parameters.

## ACKNOWLEDGMENT

The author would like to thank Dr. Laszlo Tisza of Massachusetts Institute of Technology and his colleagues Dr. Leslie L. Foldy and Mr. Robert S. Smith for helpful discussions.

[13] M. J. Klein and L. Tisza, Phys. Rev. **76**, 1861 (1949).

[14] J. Frenkel, *Kinetic Theory of Liquids* (Oxford University Press, New York, 1946). See especially p. 64.

*When light is reflected or scattered, we consider, as a rule, that it makes us see the last object from which it is reflected or scattered; when it is refracted, we consider that we are still seeing the previous source, though inaccurately. Reflected light, however, is not always taken as giving perception of the reflector; it is not so taken when the reflection is accurate, as in a mirror. What I see when I shave I consider to be my own face. But when sunlight is reflected on an outdoor landscape it gives me much more information about the things in the landscape than about the sun, and I therefore consider that I am perceiving the things in the landscape.*—BERTRAND RUSSELL, *Human Knowledge,* 1948.

# Introduction to Polarization of Electromagnetic Waves

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(Received August 21, 1950)

If microwaves are employed in the first demonstration of polarization of electromagnetic waves, then one does not need to rely upon analogy or theory to give the direction of polarization. The electric field is parallel to the antenna. Simple means are described for using a hand-sized triode transmitter and a hand-sized intensity meter in the class room demonstration and laboratory study of polarization by a parallel wire screen, polarization by reflection from dielectrics, elliptical polarization, and rotation of the plane of polarization.

HERETOFORE in the study of polarization of light in the elementary laboratory it has been necessary to rely either upon theory or analogy to give the direction of polarization. If the study is begun with microwaves, then the electric field is parallel to the antenna of the transmitter.[1] The intensity meter with its antenna serves as an analyzer. Since the current in the crystal detector is proportional to the square of the potential difference across it, the indication of the microammeter is proportional to the power received and thus to the intensity of radiation. A simple hand-sized triode transmitter and intensity meter have been described previously.[1] They are designed for 2450 megacycles/sec (12.25 cm), a frequency assigned for scientific, medical, and amateur use by the Federal Communications Commission. The oscillator and intensity meter with accessories are now available as scientific supplies. Hull[2] has described a klystron oscillator for production of 3.2-cm microwaves and its use in performing microwave experiments analogous to the common experiments in physical optics.

If the transmitter and intensity meter are aligned as in Fig. 1 and the intensity meter rotated about axis $AB$, the intensity will be found to be proportional to the square of the cosine of the angle through which the antenna is rotated.

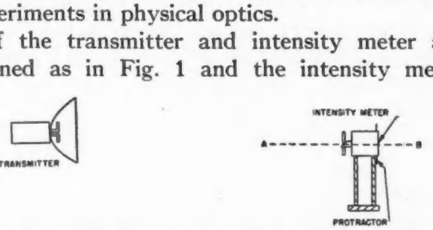



FIG. 1. Arrangement for studying polarization of microwaves.

* The devices described in this article were developed while the writer was serving as consultant to General Electric Research Laboratory, The Knolls, Schenectady, New York.

[1] C. L. Andrews, Am. J. Phys. **14**, 380 (1946).

[2] G. F. Hull, Jr., Am. J. Phys. **17**, 559 (1949).

### POLARIZATION BY PARALLEL WIRE SCREENS

A screen of parallel wires an eighth inch in diameter and an eighth wavelength or less apart may be used in the polarization of microwaves much as Polaroid is used with light. The screen should be at least 9 inches square. If the spacing between wires is too large or the wires too fine, then the wave will not be completely reflected even when the wires are parallel to the electric field. Refrigerator trays and toaster grids may serve. For qualitative demonstrations the fingers of the hand serve as "nature's own Polaroid." Strips of metal foil a quarter inch wide, separated by a half-inch and fastened to a manila folder with Scotch tape provide an excellent polarizing screen. The parallel wires can be seen better than the crystal structure of Polaroid can be visualized. It is a simple matter to compute the components of the incident electric field which are parallel to the wires and reflected and those which are perpendicular to the wires and transmitted.

The screen of parallel wires may be held between transmitter and intensity meter of Fig. 1 with its plane perpendicular to $AB$ and rotated about $AB$. If highest precision is to be obtained, the grid should be mounted over an aperture in a metal screen to avoid effects of Fresnel diffraction around the screen. As a check upon the student's understanding, the instructor may ask him to predict results if the intensity meter is set for extinction by rotating it about $AB$ until its antenna is at right angles to that of the transmitter

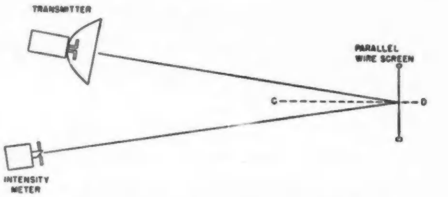



FIG. 2. Arrangement for studying reflection from a parallel wire screen.

and the parallel wire screen is inserted and rotated again. The results may be plotted on polar graph paper.

Figure 2 indicates an arrangement for studying reflection from a parallel wire screen. The screen may be rotated about axis *CD*.

### POLARIZATION BY REFLECTION FROM DIELECTRICS

The direction of polarization of light produced by reflection at Brewster's angle can be determined only by theory or by analogy. Frequently, the skipping stone analogy is used. If the flat surface of the stone is parallel to the plane of incidence, then the stone passes through the surface of the water. If the flat surface is perpendicular to the plane of incidence then it may occasionally be reflected from the surface. Such an analogy is no more than an aid to the memory.

If microwaves are employed, then the direction of polarization of the incident wave from the transmitter is known. A sheet of Transite, pressed asbestos, $\frac{1}{2}$ or $\frac{1}{4}$ in. thick by 8 in. by 15 in. may be used as a reflector. The Transite has a dielectric constant of 2.5, a Brewster's angle of 70 degrees, and a glancing angle of 20 degrees. Figure 3 shows an arrangement for studying the polarization of the transmitted beam. The sheet of Transite may be rotated about a line *EF* through its center. When the plane of polarization is parallel to the plane of incidence, the wave is totally transmitted as nearly as can be measured with the meter. When the plane of polarization is perpendicular to the plane of incidence, less than 5 percent of the intensity is transmitted. Because of the high index of refraction of Transite for microwaves and the resulting large value of Brewster's angle, the completeness of polarization of the transmitted beam is as high as for light when a stack of glass plates is used.


FIG. 3. Arrangement for studying polarization by reflection from dielectrics.

In classroom demonstrations a 20-degree plywood or cardboard wedge may be held at the edge of the Transite sheet to indicate the glancing angle. For laboratory use two such plywood wedges may be attached to the long edges of the Transite sheet as support.

This study of the polarization of microwaves incident upon a dielectric at Brewster's angle may be made in order to establish the direction of polarization prior to the study of polarization of light by reflection from glass. The light reflected from the glass at Brewster's angle may then be used to determine the direction of polarization of components transmitted by Nicol prisms and Polaroid disks. The direction of polarization is thus determined experimentally without resort to analogy or theory.

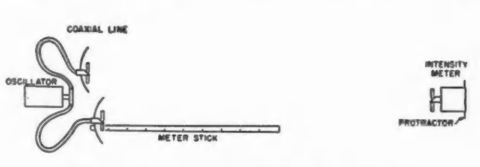



FIG. 4. Arrangement for production of elliptical polarization.

### ELLIPTICAL POLARIZATION

Microwaves provide a vivid introduction to elliptical polarization. Figure 4 shows an arrangement whereby two antennas are excited by the same source through a "*T*" and two coaxial cables of equal length, so that the the two sources are of the same phase and amplitude. This is the two-antenna adaptor used in Young's experiment in interference. The relative phases of the two waves arriving at the intensity meter are determined by path difference from the secondary sources. Figure 5 shows the two antennas set at right angles to each other and in the same plane, together with component and resultant electric fields. The intensity meter may be rotated and the intensities plotted on

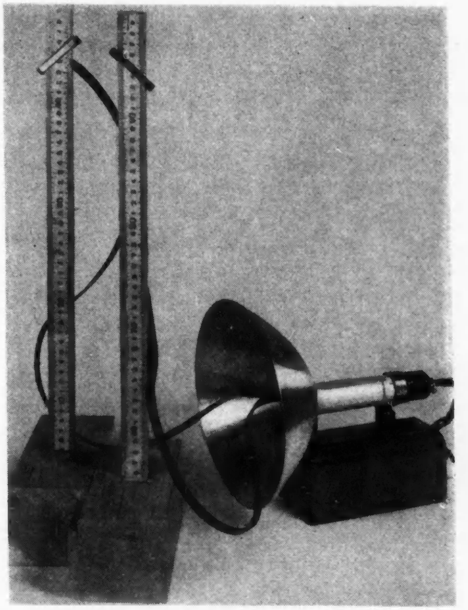
polar graph paper. The square roots of intensities or amplitudes may be plotted also and the ellipse checked with construction of the Lissajou's figure.

The two-antenna adaptor may be made with twin lead of 300 ohms characteristic impedance commercially used for television. If care is taken not to hold the twin lead in the hand or let it lie along a metal surface, it will not radiate except at the antenna. The leads to the two antennas should not lie against each other. The antennas are folded dipoles of phosphor bronze spring strip which clamps over a meter stick. The antennas are clipped over two pieces of upright meter stick as indicated in Fig. 6.

Pelsor[3] has described a wave guide analog of a half-wave plate. The relative velocity of the two waves polarized parallel to the planes of the two sides of the rectangular waveguide could be adjusted by squeezing the wave guide so that plane, elliptical, or circular polarization could be produced.

### ROTATION OF A PLANE POLARIZED WAVE

In discussing rotation of polarization by sugar solutions or quartz crystals, the instructor may drive a nut along a bolt toward the class while the class notes the direction of rotation of the nut. If he then turns the opposite end of the bolt toward the class and again drives the nut toward the class, the direction of rotation is still the same relative to the observers. The student thus observes that a molecular mechanism is conceivable that will rotate a beam of light in the same direction no matter how the molecule is oriented and that solutions of randomly oriented molecules of the proper type might rotate the plane of polarization of a beam.



Courtesy General Electric Company Research Laboratory.

FIG. 6. Twin leads to folded dipoles for production of elliptical polarization.

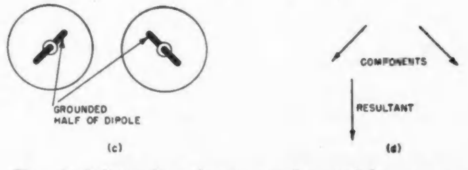
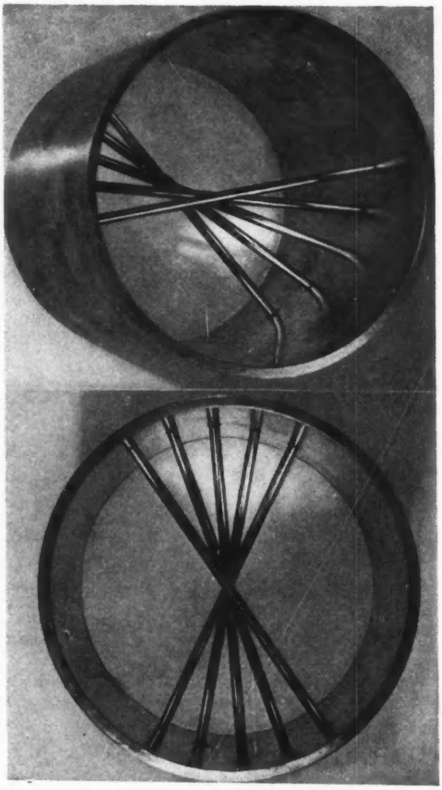



FIG. 5. Orientation of antennas for studying waves polarized at right angles to each other.

If microwaves are employed, then the individual rotator may be seen. If a parallel wire screen is set between transmitter and intensity meter of Fig. 1 with its plane perpendicular to $AB$ and the wires making an angle of 45 degrees with the electric field, then the transmitted beam is plane polarized at right angles to the wires. However, only half the intensity is transmitted. If three parallel wire screens are used with their planes separated by a half inch, the wires of the first, second, and third screen from the source making angles of 105°, 120°, and 135° with the electric field from the transmitter, then the amplitude $A$ of the transmitted beam relative to the amplitude $A_0$ of the incident beam is

$$A/A_0 = (\cos 15°)^3 = 0.90.$$

[3] G. T. Pelsor, Am. J. Phys. 17, 223 (1949).



Courtesy General Electric Company Research Laboratory.

FIG. 7. Two views of a "spiral staircase" for rotating the plane of polarization of microwaves.

If $n$ such screens are used to rotate the beam through angle $\theta$, then

$$A/A_0=[\cos(\theta/n)]^n,$$

and the intensity $I$ relative to $I_0$ of the incident beam is

$$I/I_0=[\cos(\theta/n)]^{2n}.$$

As $n$ becomes, large $I$ approached $I_0$. If a piece of cylindrical wave guide 3 or 4 in. in diameter is employed (a tomato can will do), then a one-eighth-inch diameter wire extending diametrically across the cylinder may replace each of the parallel wire screens. Figure 7 is a photograph of such a rotator designed to rotate the beam through 60 degrees. If the opposite end of the wave guide is turned toward the observer, it is still "dextrose."

## GENERAL PRECAUTIONS

The suggestions given here are for experiments that may be performed on a 3-ft×5-ft laboratory table. With the usual separation of laboratory tables of 3 ft or more, experimenters at adjacent tables will not interfere with each other, provided intensity levels are not raised above that necessary for work on one table.

Although the above demonstrations and experiments were designed for simplicity and have had repeated laboratory trial, it is the experience of the writer that many instructors prefer to devise their own microwave experiments. Since with these hand-sized waves interference and polarization effects are produced with such ease, these same effects may also be produced when they are not wanted. The reflections which are the most frequent source of error are those from the table top and the observer. If the electric field is polarized in a plane perpendicular to the table top, then the reflections will be less. Reflections from the body of the experimenter can be observed by moving the body over distances of a wavelength in a time greater than the natural period of the meter.

The precautions which have been mentioned should not deter anyone from microwave experiments. The sources of error are just enough to keep the student alert to the principles of physical optics.

The emphasis of this paper is upon the simplicity and effectiveness of using hand-sized waves in demonstrating principles of wave optics.

*In answer to a question whether I was a medium, the response was three brisk and vigorous knocks. I noticed that the knocks issued from a particular locality, and therefore requested the spirits to be good enough to answer from another corner of the table. They did not comply; but I was assured that they would do it, and much more, by-and-by. The knocks continuing, I turned a wine-glass upside down, and placed my ear upon it, as upon a stethoscope. The spirits seemed disconcerted by the act; they lost their playfulness, and did not recover it for a considerable time.*—JOHN TYNDALL, *Fragments of Science*, Vol. I, p. 448.

# Microwave Reflection from Water Spheres*

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(Received October 2, 1950)

When any material object is placed in the field of an electromagnetic wave, the object removes energy from the field. In general, some of the energy is dissipated internally as heat, and some of the energy is reradiated to produce a secondary or scattered field. The back-scattering cross section is a lumped measure of the ability of the scattering object to reradiate energy in the direction of the source.

This paper deals with the experimental determination of the back-scattering cross section for water spheres with sizes comparable with the wavelength. The determination of this parameter is fundamental in the investigation of the microwave reflection from rain.

## I. THE PROBLEM OF THE MICROWAVE REFLECTION FROM RAIN

WITH the development of microwave radar during the war, it was found that for sufficiently small wavelengths, rain could produce an appreciable echo. This echo phenomenon was important for several reasons. For the usual tactical use of the equipment it was important to distinguish between atmospheric reflections and operational targets. Also, as a meteorological tool, the observation of echoes was very useful in mapping the rain areas and in showing their movements. Thus, it is not surprising that this problem received considerable attention.

In order to investigate theoretically the microwave reflection from rain, it is necessary to assume that the drops are randomly distributed in space, and that the mutual interaction between drops is negligible. The first assumption appears to be valid without further comment. The second assumption is based on Trinks'[1] analysis that for Rayleigh scattering the mutual interaction is negligible for sphere spacings greater than two or three sphere diameters, plus the fact that in actual rain the average spacing between drops is many times this value.[2] Under these conditions the problem can be divided into two parts: (1) finding the drop size distribution, and (2) finding the reflection from a single drop. The drop size distribution is strictly a meteorological problem; the reflection from a single drop is a problem in electrodynamics.

The first thorough investigation of the reflection and attenuation effects from rain was made by Ryde *et al.*[3–7] Ryde's method was to compute the numerical results for a single water sphere, using the classical Mie[8] theory, and to use the assumptions given above to make application to to the case of many drops. It was found that if the wavelength was sufficiently long compared to the drop sizes, the well-known Rayleigh theory was adequate, while for shorter wavelengths the more exact theory was needed. Additional contributions to this problem have been made by Goldstein,[9] Haddock,[10] and others. In all cases, a major difficulty has been the evaluation of the

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1 W. Trinks, "Zur Vielfachstreuung an kleinen Kugeln," Ann. Physik **22**, 561 (1935).

2 W. Palmer, *Studies of Continuous Precipitation*, Doctoral Thesis (McGill University, Montreal, Canada, April, 1949).

3 J. Ryde, *Echo Intensities and Attenuation Due to Clouds, Rain, Hail, Sand, and Duststorms at Centimeter Wavelengths* (General Electric Company, 1941), Report No. 7831.

4 J. Ryde and D. Ryde, *Attenuation of Centimeter Waves by Rain, Hail and Clouds* (General Electric Company, 1944), Report No. 8516.

5 J. Ryde and D. Ryde, *Attenuation of Centimetre and Millimetre Waves by Rain, Hail, Fogs and Clouds* (General Electric Company, 1945), Report No. 8670.

6 J. Ryde, "The attenuation and radar echoes produced at centimetre wavelengths by various meteorological phenomena," *Meteorological Factors in Radio-Wave Propagation* (The Physical Society, London, 1946), pp. 169–189.

7 J. Ryde, J. Inst. Elec. Engr. **93**, 101 (1946).

8 G. Mie, "Beitrage zur Optik truber Medien, speciall kolloidaler Metallosungen," Ann. Physik **25**, 377 (1908).

9 L. Goldstein, "Absorption and scattering of microwaves by the atmosphere," Off. Pub. Bd., Report PB

reflection from a single sphere, especially since the functions necessary for the classical theory are not tabulated over the range needed. However, by proper manipulation of the results of classical theory, this difficulty can now be reduced greatly.[11]

A large amount of experimental work has also been done on this problem. Although most of this work has been qualitative in nature, some quantitative measurements have been made on actual rain.[12–15] These measurements show good agreement with theory considering the number of variables involved. However, since it is difficult to isolate the individual effects when there are many variables, it seems desirable to obtain some measurements on single spheres. It was with this thought in mind that the present investigation was undertaken.

## II. THE EXPERIMENTAL PROBLEM

The experimental measurement of the reflection of electromagnetic waves from individual water spheres involves several important difficulties: (1) since the sphere is a very low-gain reflector, it is hard to obtain reliable answers by ordinary pulsing techniques, and (2) it is not easy to maintain a water sphere while measurements are being made. For these reasons no direct laboratory measurements on individual water spheres have previously been undertaken successfully. Even now these difficulties almost preclude the taking of measurements in the region of greatest interest, i.e., in the spectral region where water spheres of actual raindrop size are comparable with the wavelength. However, it is possible to perform an experiment at slightly longer wavelengths and to increase the size of the water spheres to get into the critical-size region. This is the experiment actually performed. It is believed that the technique is useful, since comparison between experiment and theory at one frequency should be a good indicator of expected comparisons at other frequencies.

*Method of Measuring the Back-Scattering Cross Section.* Although it is possible to measure the back-scattering cross section $\sigma$ by direct radar intensity measurements, it is usually not practical as a laboratory method. Direct intensity measurements are subject to the usual inverse fourth-power range attenuation which may be quite a handicap for low-gain scatterers. Also, pulsing techniques are difficult to apply at the short ranges which most laboratories will allow. Therefore, this method was ruled out from the start.

Of the several laboratory methods which might have been utilized, the one actually used was the standing-wave method of King.[16] This method utilizes the analogy between wave propagation in space and on a transmission line: the far-zone field of an antenna is a spherical traveling wave with the same propagation constant as on a lossless transmission line. An object placed in the path of this traveling wave removes some energy from the wave and reradiates a secondary field. The interaction of the incident and reradiated fields causes standing waves to be set up in space. By analogy with the transmission line case, a reflection coefficient, which is simply related to the back-scattering cross section, can be defined in terms of the standing waves. The resulting expression is more complicated than for a transmission line. The free-space wave has an amplitude decay proportional to the inverse first power of the distance from the source, and this causes the standing-wave ratio to be a function of its position in space. However, formulas of practical value can be developed, subject to only mild restrictions. These are discussed in the literature.[11,16]

5850, NDRC Report WPG-11 (1945); also published in *Radio Wave Propagation* (Academic Press Inc., New York, 1949), Vol. II, Chap. 5.

[10] F. Haddock, "Scattering and attenuation of microwave radiation through rain," paper presented at the joint meeting of the International Scientific Radio Union and the Institute of Radio Engineers, held at Washington, D. C., (1947).

[11] A. Aden, *Electromagnetic Scattering from Metal and Water Spheres* (Cruft Laboratory, Harvard University, 1950), Technical Report No. 106.

[12] H. Goldstein, *The Effect of Clutter Fluctuations on* MTI (Radiation Laboratory, Cambridge, Massachusetts, 1945), Division 14, Report 700.

[13] Marshall, Langille, and Palmer, J. Meteorol. **4**, 186 (1947).

[14] R. Langille and K. Gunn, J. Meteorol. **5**, 301 (1948).

[15] J. Hooper and A. Kippax, Proc. Inst. Elec. Engrs. **97**, 89 (1950).

[16] D. King, *Measurement and Interpretation of Antenna Scattering* (Cruft Laboratory, Harvard University, 1948), Technical Report No. 50; also published in Proc. Inst. Radio Engrs. **37**, 770 (1949).

With the method outlined above, it is possible to determine the back-scattering cross section of a scattering object by measuring the magnitude and position of the standing waves. In theory, it is feasible to think of actually detecting the standing waves in space. However, in practice this procedure is not too practical, since with a low gain scatterer the detecting apparatus and the observer might generate larger standing waves than the object being investigated. Therefore, an image screen technique is used.

With the image screen technique, the scattering object is divided along a plane of symmetry, and one-half of the object to be measured (in this case, a hemisphere) is mounted on a large, highly conducting screen. The standing waves set up on the screen by the interaction of the incident and reflected waves are measured along the line between the source and scatterer. Measurement is accomplished by projecting a small signal probe through a slot in the screen. By the theory of images,[17] the measurements taken are the same as for the complete scattering object in free space. The advantages of the image screen technique are: (1) it simplifies the problem of supporting the scattering object and the test probes, and (2) by placing the observer and his equipment behind the screen, stray radiation to the measuring equipment is minimized and movements of the observer have no effect on the measurements.

The standing-wave method offers the advantages of a system having absolute calibration, relatively simple equipment, and measurements at low power. The main disadvantages are that it requires an image screen which is large, uniform, and rigid, and that the obtainment of results is very time consuming.

Schematic representations of the two methods mentioned above are shown in Figs. 1 and 2.

*Method of Maintaining the Water Spheres.* One of the worst obstacles to making measurements on water spheres is the inability to maintain such a sphere while measurements are being taken. One procedure for overcoming this drawback (employing the image technique) is to use as a

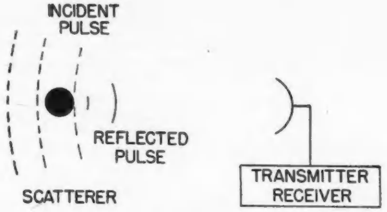


FIG. 1. Direct radar intensity measurements -- pulsed operation.

container a thin hemispherical shell of low dielectric material fastened to the image screen. The water is contained between the shell and the image screen. Since the relative dielectric constant of water is so high, the only effect of the shell (provided it is thin enough and its dielectric constant low enough) is to increase the transition zone between the water and air.

In the present investigation, the above technique was utilized. The thin hemispherical shell forms were machined from Styrofoam[18] and mounted on aluminum disks. Since Styrofoam has dielectric properties extremely close to those of air, it had a negligible effect on the measurements. In fact, large pieces of it may be moved in the vicinity of sensitive probes without noticeable effect. This is true even with electromagnetic waves of 1-cm wavelength.[19]

*Description of the Equipment.* A block diagram of the experimental setup is shown in Fig. 3. The image screen, made of aluminum sheeting, was a square of about 70 wavelengths on a side. The transmitter was a continuous-wave oscillator operating at a wavelength of 16.230 cm and

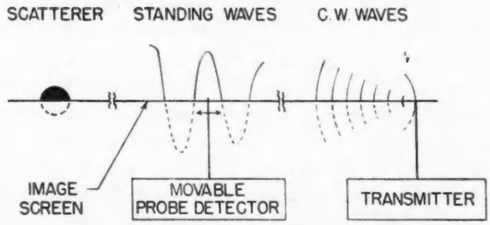


FIG. 2. Standing-wave method, C. W. operation. The diagram depicts the decay of the standing-wave amplitudes with distance from the scatterer. The distance between significant points is compressed on the diagram as indicated by drawing breaks in the screen.

[17] R. King, *Electromagnetic Engineering* (McGraw-Hill Book Company, Inc., New York, 1945), Vol. 1, Chapter IV, Sec. 20.

[18] Product of The Dow Chemical Company, Midland, Michigan.

[19] R. Kodis, Doctoral Thesis, Harvard University (1950).

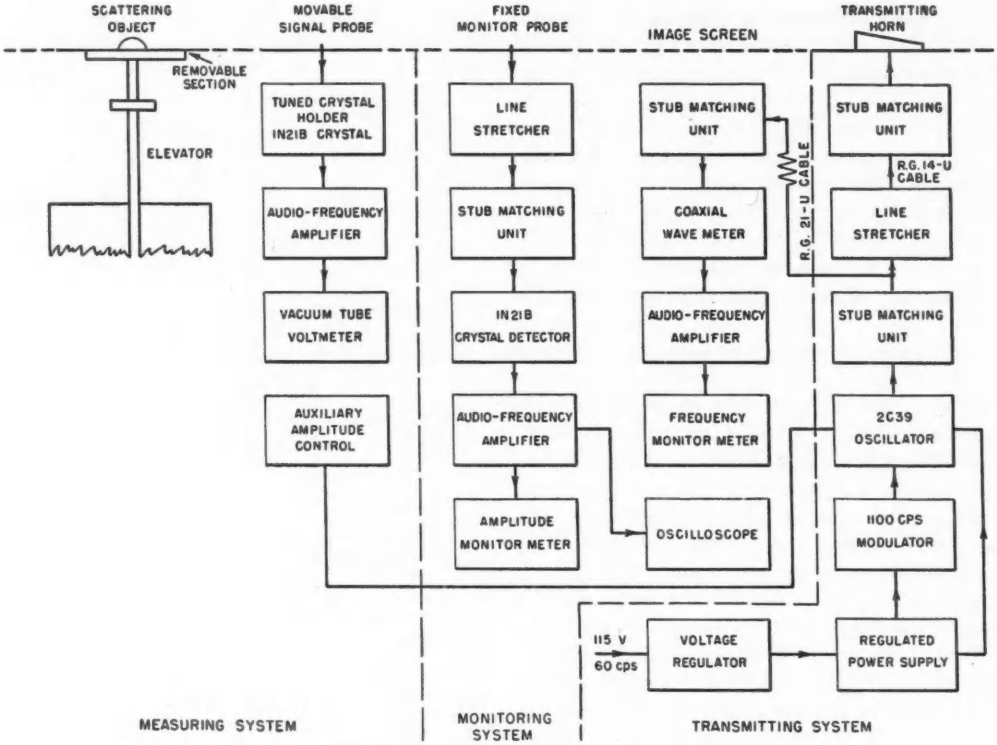


FIG. 3. Block diagram of the experimental equipment.

modulated at a frequency of approximately 1100 cycles/sec. The transmitting antenna was a sectoral horn located at one corner of the image screen and directed diagonally across the screen. The opening through which the scatterers were extended was located about two thirds of the way across this diagonal. The distance between source and scatterer was thus about 59 wavelengths. Suitable mechanical means were used to mount the scattering objects and probes.

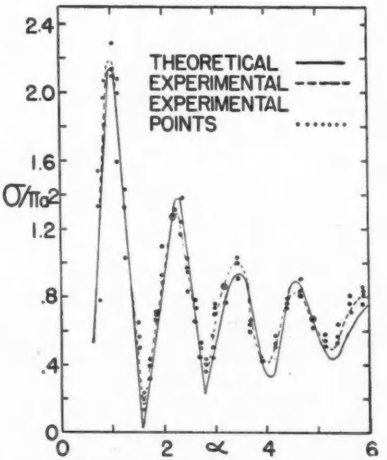


FIG. 4. Back-scattering from water spheres.

## III. EXPERIMENTAL RESULTS

Measurements were made on thirty water spheres in the critical electrical-size region given by $0.74 \leq \alpha \leq 5.90$, where $\alpha$ is the electrical sphere size given by $\pi$ times the ratio of the sphere diameter to the wavelength. The results of these measurements are shown in Fig. 4. Here, the back-scattering cross section, normalized with respect to the geometrical area of the sphere, is plotted against the electrical sphere size $\alpha$. The corresponding theoretical curve is shown for comparison. The method used for obtaining the theoretical curve is discussed elsewhere.[11]

It is believed that these were the first experimental measurements of the back scattering from individual water spheres in this critical size region. It is seen that there is very good agreement between the experimental and theoretical results. For spheres with sizes very much smaller than those considered here, the results would be given to a good approximation by the Rayleigh law. For spheres with sizes larger than those considered here, the results would approach the large-size approximation given by the solid horizontal line at the right hand side of Fig. 4.

**ACKNOWLEDGMENTS**

The author gratefully acknowledges the guidance and encouragement of R. W. P. King during the course of the investigation. Thanks are due N. C. Gerson for reading the manuscript.

## The Chronology of Modern Physics*

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The present day student of physics will be called upon to interpret his field to the world at large as a force in the contemporary social, political, and economic scene. The insight required to do this cannot be gained without a knowledge and understanding of the history of physics. Standard treatments, even when they include separate "History of Physics" courses, fail to provide the necessary integration. It is suggested that the area of modern physics offers an opportunity for organization along modified chronological lines such as to include along with rigor the bulk of the history of physics. Several advantages from the standpoint of logic and pedagogy are pointed out.

PHYSICS has rather recently come of age in the sense that it is being recognized as a force in the social, political, and economic world. Our colleges are recognizing that some acquaintance with its methods and ideas should be part of the cultural development of all our students. That these methods and ideas are being widely diffused beyond the close circle of physicists is a good and healthy thing. It has already destroyed much of the "ivory tower" illusion about physics research and drawn physicists into the center of social, political, and economic controversy. I am concerned about the breadth of understanding that our students, the physicists of the future, who will step into this situation immediately on graduation, will bring with them to the contemporary scene. The questions they will be asked most often are "What can physics do?" and "Where is physics going?" Can they attempt an answer without knowing the answers to "Where is physics now?" and "Where has physics been?"

Scanning the traditional physics curriculum as evidenced by the majority of standard texts, I find little that will help them answer these questions. These texts are largely a jumble of assorted topics, great discoveries, and a few famous names and dates, liberally sprinkled with carefully fabricated examples of the scientific method. Worst of all, each author has taken the liberty, to a greater or lesser degree, of reconstructing physics to fit a pattern of logic which appeals to him, often with the obvious purpose of presenting a watertight, pat argument to which he can append a capital "Q.E.D." Thus the history of physics is given a warped presentation, the present appears to hold all that is worth knowing, and there is no challenge for the future.

Physics has a history of its own, a history to be proud of, a history which every physicist should know. This history is not an accident, nor is it something apart to be relegated to a separate

* Address delivered at meeting of the Oregon Section, May 13, 1950, at the University of Portland, Portland, Oregon.

† Present address: Armour Research Foundation of Illinois Institute of Technology.

course labeled "pure culture." Furthermore, inconsistencies of logic do not occur in history with the frequency at which they appear in textbooks.

In the broad, basic physics courses where the student is acquiring the elementary tools of his trade, the historical approach developed to any great degree would probably prove too slow and cumbersome. In this area I have had no experience. In specialized courses, where modern methods have superseded, condensed, or recrystallized the historical argument, perhaps a minimum of what then becomes "ancient history" is in the interest of time economy. However, on the basis of personal experience, it appears to me that in a course in modern physics we have a wonderful opportunity to present not only the facts and methods, but the history. Furthermore, since modern physics draws on all areas of classical development, the historical framework can be essentially complete. My thesis is that an historical structure can be used in building such a course with no sacrifice as to the amount of material covered, the rigor with which it is presented, or the mathematical logic of its structure. Rather, in my experience, what I shall call a modified chronological development provides a definite improvement in logical structure and student response over traditional treatments.

The end result of such an approach, I submit hopefully, should be not only better general education for physicists but better physics training as well. How an idea arose is often as important as the idea itself. The so-called elementary particles are constructs whose claim to reality is on a far different basis than that of a chair or lecture table. A review of the more than a century of experiments, hypotheses, and theories from the first experiments on electrolysis, through cathode rays, the Edison effect, the work of Hertz, Lorentz, Thomson, and then Millikan leads to a far more mature understanding of this difference than a newly revised text claiming many adoptions that starts the first page of Chapter I with "Millikan's Oil Drop." Furthermore, history presents the dynamics of physics, where theories can be corrected and modified on the basis of new evidence or thrown out entirely in favor of brave new hypotheses which prove more powerful. A concept of approximations to the ultimate goal of truth which may never be reached is substituted for the pat assurance of a text which "tells all."

A complete chronological treatment is not desirable. What I call a modified chronology is an historical development of the formulation of each of perhaps a half-dozen fundamental ideas or theories which can be carried along independently up to the point where they are in shape to be merged with or incorporated into the development of a subsequent idea or theory. The selection and ordering of these "fundamental ideas or theories" is based on logical synthesis rather than strict chronology, and the introduction of each of the several sections means backtracking in time; hence, the "modified chronology."

Such a framework, which is certainly not unique, but which I have found quite successful, is the following. In use, I have supplied my students with an eleven-page mimeographed set of notes in which each of the following seven topics is expanded by listing with some discussion in strict chronology a set of significant advances in its area:

*Section 1.* "The Atomic Nature of Matter" pays brief respect to the ancients, develops the essentials of kinetic theory as proposed by Bernouilli, discusses the work of Proust, Dalton, Berzelius, Avogadro, Prout, and Newlands, and culminates in the periodic table of Mendelejeff and Meyer.

*Section 2.* "The Electrical Nature of Matter," with the alternate title "The Discrete Nature of Electricity" briefly mentions the history of static electricity and traces carefully the developments in electrolysis, discharges in gases, cathode rays, thermionic and photo-effects, the theory of Lorentz, and the classical experiments of Thomson and Millikan, with constant evaluation of the evidence and its possible implications at each step.

*Section 3.* "The Origin of Quantum Theory" section begins with blackbody radiation, with Wien, Lummer and Pringsheim, Lord Rayleigh, Max Planck, thence to Einstein's work on photoelectricity and specific heats, with a foreshadowing of the work of Bohr.

*Section 4.* "X-Rays" are treated in a short section, tracing the development from Roent-

gen's discovery through the work of von Laue, Bragg, and Moseley, and contribute to the quantum idea through the law of Duane and Hunt and the Compton effect. The wave-particle complementarity is further strengthened by the work of De Broglie and Davisson and Germer.

*Section 5.* "Radioactivity and the Nuclear Atom" treats radioactivity in full with an accurate presentation of the experiments which led to the identification of the radiations, the theory of atomic instability of Rutherford and Soddy, and the discovery of isotopes through the chemistry of Soddy and Boltwood. The finale of this section is the scattering experiments of Geiger and Marsden and their explanation by Rutherford.

*Section 6.* "Atomic Spectra and Atomic Structure" traces the development from Newton's first work with prisms through many topics which can be covered quickly if the course is preceded by work in optics. The work of Kirchoff, Rowland, Balmer, and Rydberg gets special attention; the Bohr theory of the hydrogen atom is presented in full; and the tie-in with x-ray spectra is established. The degree of extension into further developments of atomic spectra and structure has varied with the time available and the interests of the class.

*Section 7.* "Nuclear Transmutation and Nuclear Structure" is prefaced by discussion of Einstein's $E=mc^2$, and proceeds from Rutherford's historic transmutation of nitrogen to the present in strict chronology as completely as time permits.

These seven topics would not be sufficient for an all-embracing course in modern physics, but have been specifically developed for a one-semester course in atomic and nuclear physics. I am sure the notion of a modified chronological treatment which I have tried to illustrate can be applied successfully to the specific demands of any course in the modern physics area. Certain by-products of the approach are worthy of mention:

Recapitulation of material the student has covered elsewhere belongs, I think, on the positive side of the ledger. It can be outlined in the same detail as new material, but class time devoted to it can be cut to a minimum. It ties in the present argument with what the student already knows and helps orient what may be special islands of knowledge in the scheme of things. This is good pedagogy.

In my experience, it turns out that the chronological order of things is, after all, as good a logical order, if not better, than that which has appealed to many authors. The chronological nature of the presentation automatically avoids the necessity of such statements frequently recurring in texts as "this is all we need to know about this now," as one finds with reference to the alpha-particles when an author feels called upon to discuss the nuclear atom seven chapters ahead of his discussion of radioactivity. In the same vein, it avoids the frequent references to later topics, which leave the student up in the air with gaps in the logic which he will probably not fill in if and when he comprehends the later section.

Best of all, the chronological development of ideas avoids the impression of false historical order stamped in the student's mind by inversions made in the name of "logic." Such an inversion is not put right by the author's admitting his guilt. The order of presentation will stick in the student's mind long after the apology is forgotten. The drama of human endeavor in the unravelling of nature's secrets is more likely to appeal to the student than the perhaps more straightforward dry logic of a fabricated order of things. To my way of thinking, placing Millikan's oil drop experiment at the beginning of a course in modern physics is like printing the last chapter of a detective story first.

To those who have shared my dissatisfaction with available texts, let me recommend sitting down with Cajori's *A History of Physics*, Crew's *Rise of Modern Physics*, and other source material and exploring the possible advantages of an historical approach.

# Elementary Laboratory for Premedical Students

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(Received June 27, 1950)

A brief description is given of supplementary experiments for premedical students, with a bibliography of useful references. Included are: the back as a lever, with calculation of the force on the fifth lumbar vertebra; charting of temperature and pulse rates for a considerable period; a metabolism test with a calculation of the rate of energy production in the body: individual audiometer tests following a lecture on hearing; the physiology of the eye: formation of images on the retina; the relation between acuity and illumination: color and theories of color vision; a discussion of the problem of nerve conduction; the use of a Wheatstone bridge as a lie detector; electrocardiographs. It seems successful as judged by the caliber of the students' work and by their interest.

FOR several years we have been trying the experiment of running a separate laboratory section for the premedical students studying elementary physics. They are given two instead of the usual one laboratory period a week and for this they receive an extra hour of credit each semester. The plan seems to be successful both in its economy of staff, as the lectures and recitation-demonstrations are the same for all the elementary students, and in its stimulation of the interest of this specialized group. The difficulty in planning has been chiefly that of finding material sufficiently simple so that the students can work intelligently, not cookbook fashion, and at the same time see the connection between this work and that of their primary field.

The laboratory work has been closely correlated with the prescribed textbook; as we changed the latter, the order has been changed. In Table I are listed both the regular experiments and the additional experiments or lecture-demonstrations. A very considerable number of the added experiments would be done by all of the students if we had time for them; they have no particular medical "flavor" but seem essential. We make no claim to originality in the new experiments, but feel that, since the necessary references[1] and materials are in some cases unfamiliar to a physics group, a brief description is warranted.

In "The back as a lever," the actual measurements are quite simple. Each student weighs herself on bathroom scales, and a fellow student measures the length for her, from the fifth lumbar vertebra to the seventh cervical where the effective weight of head and arms may be considered suspended when she bends over (Fig. 1). Assuming[2] that, in leaning over at an angle of 60° the suspensor muscle makes an angle of 12° with the trunk and that it is attached about $\frac{2}{3}$ of the distance along the trunk, that the load of head and arms is 20 percent of the body weight, and the trunk is 40 percent of the body weight, the problem is to find the compression exerted on the fifth intervertebral disk. The results are enlightening in their magnitudes and in the "skewness" of the resultant, and the solution of the problem involves a fairly stiff workout in resolution of forces as well as in torques. It is always pointed out to the group that the results are valid only for an order of magnitude, even though their measurements seem much better than that. The muscle attachments are not at one point but are spread over an area; the angles given are the results of experiments on other individuals and cannot be checked as far as their applicability here is concerned; the proportion of a student's total weight assumed as effective at the points used can be no better than an estimate. In spite of these qualifications, the ex-

[1] Physiology references used throughout the year are: for elementary discussion, H. E. White, *Modern College Physics* (D. Van Nostrand Company, Inc., New York, 1948); A. Carlson and V. Johnson, *The Machinery of the Body* (University of Chicago Press, Chicago, Illinois, 1947); for more detailed treatment of certain topics W. Howell (Edited by V. F. Fulton), *Textbook of Physiology* (W. B. Saunders Company, Philadelphia, Pennsylvania, 1946); D. Glasser, editor, *Medical Physics* (Year Book Publishers, Chicago, Illinois, 1944).

[2] Data from Strait, Inman, and Ralston, Am. J. Phys. 15, 375 (1947).

periment seems worth while because of its physiological aspects and implications.

The thermometer experiment is our solution of the problem posed by the fact that our students possess no more than the slightest of introductions to the study of heat at the time when laboratory work in that subject begins. It was inspired by sheer curiosity on the instructors' part concerning the magnitude of the temperature variations that occur in a normally healthy individual. Each girl is given a clinical thermometer and checks its lag and reproducibility by using it to measure the temperature of water in a thermos bottle. She then checks the reproducibility of readings of her own temperature. She is asked to record her temperature and pulse rate regularly three times a day for three or more weeks, and irregularly before or after unusual experiences, be they quizzes, dates, or a basketball game. The results have intrigued all of us. Some records have been kept and charted for several months.

For a relation between exercise and the rate at which heat is developed by the body, we repair to the infirmary for a metabolism test. From the amount of oxygen consumed by the body in a given time (measured and then reduced to standard conditions) and the heat of combustion of carbon, the number of calories used per hour is determined. If the rates are taken before and after some fairly strenuous exercise, such as jumping, the increase is very marked. Perhaps more than any other of our experiments, this one ties together, in an intimate way, the relation between the language and methods of the physics and the physiology laboratories.

The experiments in electricity follow a very usual order: static electricity, Ohm's law, electrolysis, the potentiometer, and the Wheatstone bridge. To this we have added one on the conduction of electricity by a "current sheet." With this as a background, the lecture on the physiological aspects of the subject can introduce the students to an astonishing variety of interesting material, the conduction of electricity by nerves and the "all or nothing" law;[3] the electrical resistance of the human body with its surface effects; the variation of its resistance with emotion and the resulting possibility of using a Wheatstone bridge as a lie detector; emf's produced by muscular action and their application in the electrocardiograph; brain waves and their distortions in epilepsy; cautionary warning of the dangerous or possibly lethal effects of electric

TABLE I. Elementary physics experiments.

| Mechanics, heat, and sound |
|---|
| Slide rule. |
| Instruments and their accuracy. |
| Parallelogram of forces. |
| *Archimedes principle. |
| *Specific gravity: liquids, bone, etc. |
| Inclined plane of Galileo. |
| *Precision and probable error. |
| Newton's second law of motion. |
| Equilibrium, parallel forces. |
| *The back as a lever. |
| Boyle's law. |
| The pendulum. |
| |
| *Temperature and pulse rates. |
| Specific heat and heat of fusion. |
| *Metabolism. |
| *Surface tension—lecture. |
| *Surface tension—laboratory. |
| Resonating columns. |
| *The ear—lecture. |
| *Audiometer. |
| **Light** |
| Photometry. |
| Reflection and refraction, plane surfaces. |
| Reflection and refraction, curved surfaces. |
| Lenses and their uses. |
| *The eye—lecture. |
| *Eye-model, geometrical images. |
| *Visual acuity and illumination. |
| Diffraction. |
| Spectrum analysis. |
| *Color vision—lecture. |
| *Color vision—laboratory. |
| **Electricity** |
| Static electricity. |
| Electrical circuit components; Ohm's law. |
| Electrolysis. |
| Potentiometer. |
| *Wheatstone bridge. |
| Magnetic fields (2). |
| *Current sheet. |
| *Induced emf. |
| *Thermocouple. |
| *Nerve conduction—lecture. |
| *Lie detector. |
| Resonance, ac series. |
| Triodes, static characteristics. |
| *Triodes, dynamic characteristics. |
| *The electrocardiograph. |
| Radio receiving set. |
| *CRT and CRO. |
| Radioactivity. |
| *Medical applications of radioactivity. |

* Experiments done by premedical students only.

[3] In addition to references listed in footnote 1, see H. S. Gasser, J. Appl. Phys. 9, 88 (1938) and O. Stuhlman, *Introduction to Biophysics* (John Wiley and Sons, Inc., New York, 1943).


FIG. 1. The back as a lever.

discharges, before a mention of shock therapy.[4] In the light of such possibilities the laboratory experiments seem perhaps too simple. The use of a Wheatstone bridge as a lie detector involves, as additional apparatus, simply made electrodes —thin and flexible metal strips with soldered wire for connection to the bridge, and a tube of electrode paste. The metal strips are fastened to the forearms with elastic bands, after the metal has been coated with the paste. The electrocardiograph is taken at the infirmary. We have found our technicians both willing to cooperate and clear in their explanations.

The desirability of a working knowledge of vacuum tube circuits seems obvious. The few experiments included cannot pretend to give more than an introduction, but at least the underlying theory and some of the possibilities are met in the electronics experiments. The work on the triode concerns both static and dynamic characteristics. The breadboard model of the cathode ray oscilloscope uses the 1-inch RCA tube (No. 913) in the set as designed by Dr. Lion of Massachusetts Institute of Technology,[5] but with the addition of a sweep circuit (see Figs. 2(a) and (b)). With wiring back of the board and students in front and with a fuse in the power supply, the use of the 450–500 volts needed for acceleration does not seem to involve undue risk. The use of this simplified version is followed by an introduction to several of the standard forms of the CRO.

Studies of the eye and of the ear offer perhaps the best applications of physics to the understanding of the working of the body. A lecture on the physiology of the ear and the physics of hearing is given in one period;[6] in the next, each student uses an audiometer. This consists primarily of apparatus borrowed from the electronics laboratory—an audio oscillator, attenu-

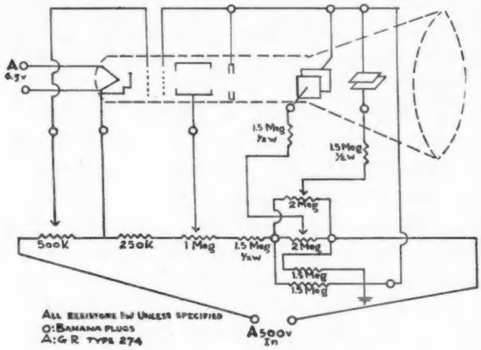


FIG. 2a. Cathode-ray tube circuit by K. S. Lion.

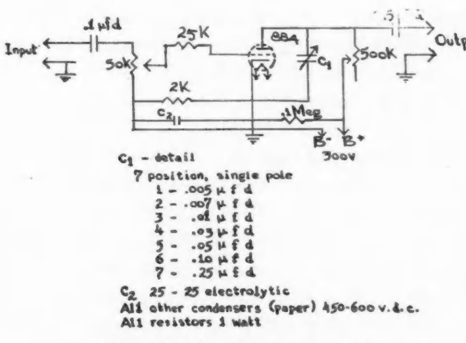


FIG. 2b. Linear sweep generator.

[4] J. D. Goodell, "Electro-medical diagnosis and therapy" and F. Offner, "Brain wave records in medical diagnosis," reprints furnished by Offner Electronics, Inc., Chicago; H. Hoagland, Science **109**, 157 (1949).

[5] K. S. Lion, Am. J. Phys. **15**, 161 (1947).

[6] S. S. Stevens, *Hearing: Its Psychology and Physiology* (John Wiley and Sons, Inc., New York, 1938); E. G. Wever, *Theory of Hearing* (John Wiley and Sons, Inc., New York, 1949).

ator, matching transformer, and calibrated ear phones (see Fig. 3). The experiment should be carried out in a soundproof room; lacking that, we have students come to class in the evening, for quiet is essential. The experiment involves variation in intensity for each frequency tested; either the attenuation is increased until the sound is lost, or starting with very low power the volume is increased until the sound is barely audible. For those quite deaf in one ear, this method is useless on account of bone conduction.

As soon as the simple principles of lenses have been mastered, the lens system of the eye is studied by using a well-known eye-model.[7] This experiment is followed by a lecture on the physiology of the eye. The discussion includes a calculation of the limit of acuity set by the retinal structure. The experiment that follows links this topic with the photometry experiment by a study of the relation between acuity and illumination. A printer's sample sheet, with the lines printed in the same kind of type in different sizes, is mounted on a black card and placed about 6 ft away from a student in a dark room. The illumination can be varied either by connecting a lamp to a Variac or by using a diffuse source and screens with openings of different areas. The former is very simple, but the fact that there is no check on the possible effects of the color changes should be pointed out. Starting with a low voltage or a small window area, one student reads the resulting illumination with a foot-candle meter (photoelectric meters of the usual sort are inadequate under low levels of artificial light) while the other reads the finest line of type possible. The graph of illumination against type size usually shows the break due to the change from cone to rod vision. The relation between possible acuity and the brightness of a fluoroscopic screen on which details are to be observed is called to the students' attention.

The subject and data of color vision are so varied and, as far as theories are concerned, inconclusive[8] that it is a temptation to ignore them in our course, but their fascinations are too great! In the year 1949–50, we followed a lecture

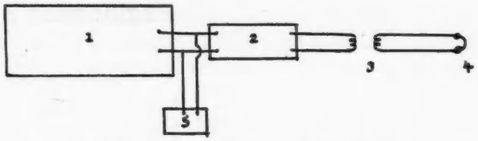


FIG. 3. The audiometer. (1) Audio oscillator, with flat response for a considerable range, such as Hewlett Packard Model 202 D; (2) Attenuator—Hewlett Packard Model 350 A; (3) Matching transformer UTC, LS-33; (4) Calibrated dynamic phone PDR 10 (Permaflux Corporation, Chicago, Illinois), headphones with 1 dummy phone; (5) Ballantine vacuum-tube voltmeter.

on color and color vision by an observation-type experiment. Subtractive and additive color mixtures were observed using standard devices as well as the prism and concave mirror combination described by White (reference 1, p. 437). Conditions for monocular fusion were illustrated by a Lumière or Agfa mosaic color picture, viewed directly and through a microscope. Binocular fusion was observed with Hecht's device.[9] This is a box about 6×8×18 inches, painted black inside. One end is left open, and in the other end two filters (such as Wratten No. 29 and No. 58)[10] are placed so that one eye looks through the red while the other looks through the green filter. With a point source of light at the open end of the box, fusion is complete for most observers. Such material as this, in which lectures, reading, and observations are to be correlated, is written up as an essay-type discussion.

An experiment in radioactivity has been introduced for all students. In it they use a "Radioactive Demonstrator"[11] and nuclear track plates[12] impregnated with radiothorium. The final paper assigned to the premedical group is on medical implications of radioactivity. The results show maturity and interest as well as great variety.

The course as outlined is subject to change almost without notice. Nevertheless, we feel that we have achieved a type of work that stimulates interest and correlates the subjects of physics and physiology. Doubtless, the correlation is capable of being greatly improved. We are open to suggestions!

[7] Central Scientific Company, Chicago, Illinois.

[8] D. L. MacAdam, Phys. Today **1**, 10 (1948). H. Hartridge, Science **108**, 395 (1948). S. Hecht, Am. Scientist **32**, 159 (1944), and J. Opt. Soc. Am. **20**, 231 (1930); **21**, 615 (1931).

[9] S. Hecht. Proc. Natl. Acad. Sci. **14**, 237 (1928).

[10] Eastman Kodak Company, Rochester; data in the Eastman *Wratten Filter Book*.

[11] Tracerlab, Boston, Massachusetts.

[12] Heinicke Instrument Corporation, Rochester, New York.

# Engineering Physics at Cornell*

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The establishment, administration, and conduct of a new program of technical training to prepare students for industrial research and advanced engineering development at Cornell University is described.

A NEW program of technical education and training was formally instituted at Cornell University in 1946. Its objective was to provide interested students with a more suitable and effective training for industrial research and advanced engineering development than could be obtained from traditional curricula in science or engineering. The program consists of a five-year undergraduate curriculum which has been given the rather undescriptive name of "Engineering Physics" leading to the degree of Bachelor of Engineering Physics. This can be followed by graduate work at the master's or Ph.D. level. In this discussion, the emphasis will be placed mainly on the undergraduate program in engineering physics though the graduate operations are deemed necessary and important for the stimulation and nourishment of the undergraduate program.

The creation of the engineering physics program at Cornell was an interesting and unusual operation in itself. It began with the feeling, later leading to conviction, on the part of faculty members of the university who during the war had an intimate contact with research and development programs, that a better technical background and training could be provided for those engaged in these fields than that normally available through traditional, somewhat compartmentalized, curricula. It was found that this view was shared by the Dean of the College of Engineering, Dr. S. Hollister, by the Dean of the College of Arts and Sciences, Dr. C. W. deKiewiet, and by the President of Cornell University, Dr. Edmund E. Day. With so much unanimity of thought and opinion it was possible to do a rather rare thing in educational circles, namely, to design an entirely new educational training program which would best meet the objective mentioned above without regard to existing division lines between schools, departments, or colleges, without undue regard to existing courses, and without regard to associated administrative problems. It was in this atmosphere and a close association with many leaders in industrial and government research that the engineering physics program at Cornell was born and since, modified and developed. To many of us it is one of the most interesting of educational experiments and at present it holds high promise of success.

## GUIDING PRINCIPLES BEHIND THE ENGINEERING PHYSICS PROGRAM

In order to appreciate fully the nature of the engineering physics curriculum and how it achieves its objective, it is necessary to understand the principles which guided those who created it and which govern the manner in which it is now conducted. These principles were arrived at by carefully considering the actual nature of the work many industrial research workers are called upon to do and just where the training supplied by traditional curricula failed to provide the most suitable training, both technically and psychologically. As background I should like to contrast in very brief terms what we regard as characteristic of the training and function of the professional engineer and that of the scientist.

The objective of the training of a scientist is to provide him with a thorough understanding of the laws and principles which account for the behavior of the phenomena of nature and a commanding analytical skill which will assist him in making further discoveries or additions to fundamental scientific knowledge.

* An invited paper given before the New York State Section of the American Physical Society at Clarkson College of Technology, September 30, 1950.

The objective of the training of the engineer has come to be that of providing the means for him to make immediate application of well-established and tested scientific knowledge. This is reflected in more detail by Dean Hollister of Cornell, who states that an engineer must be trained "to apply creatively scientific principles to design or develop structures, machines, apparatus or manufacturing processes, or works utilizing them singly or in combination; or to construct or operate the same with full cognizance of their design and of the limitations in behavior imposed by design; or to forecast their behavior under specific operating conditions, all as respects an intended function, economies of operation, and safety to life and property."

Over-concisely the scientist might be characterized by the words *understanding* and *analysis*, and the engineer by the words *application* and *synthesis*. It is clear from this that the trainings involved in the two cases are very different. The two types of training are useful in industrial research, but, on account of the fact that in modern industrial research many workers are involved in work of a pioneering character which may be years ahead of technological know-and where well-tested scientific information does how not exist or even appropriate materials, it seemed to us that the most appropriate training would, among other things, combine certain features of both types of training in *one and the same individual*. It is also required that the industrial researcher must possess a thorough knowledge of the constitution and properties of modern materials. We therefore set out to provide a new unified training which would enable the graduate to be an engineering physicist and not an engineer with a science option or a scientist with an engineering option.

With these considerations in mind we can now briefly state what we consider the objective of our engineering physics curriculum to be. It is to provide a type of training which thoroughly integrates the basic scientific knowledge and analytical background of the physicist, and to a certain extent the applied mathematician, with an adequate knowledge of the physical constitution, chemistry, and properties of available materials and the technological practices and methods of the engineer.

It will be recognized that to achieve this objective it is essential in this program that the typical barriers that tend to divide the province of pure science and engineering into separated departments of physics, mathematics, chemistry, and schools of electrical, mechanical, chemical engineering, etc., be erased. Before discussing the actual curriculum, I would like to indicate how this fusion into a unified whole has been realized.

### FACULTY AND ADMINISTRATION FOR ENGINEERING PHYSICS

The high degree of integration of subject matter and general point of view has been facilitated by the administrative organization, the choice of faculty, and the care taken in advising students.

A carefully selected faculty together with a director constitute the Department of Engineering Physics. Administratively this department is placed within the College of Engineering and the Director has the same status as the director of any of the other schools of engineering such as electrical, mechanical, etc. To insure full responsibility of the College of Arts and Sciences for the unified character of the program, the director of the Department of Engineering Physics is also the Chairman of the Department of Physics, which is administered by the latter college. In this way the Deans of Engineering and Arts and Sciences both have a responsibility for the successful operation of the program. Thus far this type of administration has been very successful.

In the interest of obtaining a high degree of integration of subject matter, members of the faculty of the department are, for the most part, carefully selected representatives of the important fields of study no matter to what college or department they belong. Consequently, the faculty is composed of members of several colleges and many departments. With few exceptions faculty members remain firmly attached to the normal departments of their professional interests. Consequently, the faculty is composed of representatives of departments such as physics, mathematics, chemistry, and the like, and various schools of engineering who are especially qualified because of industrial experience and interest in this type of education.

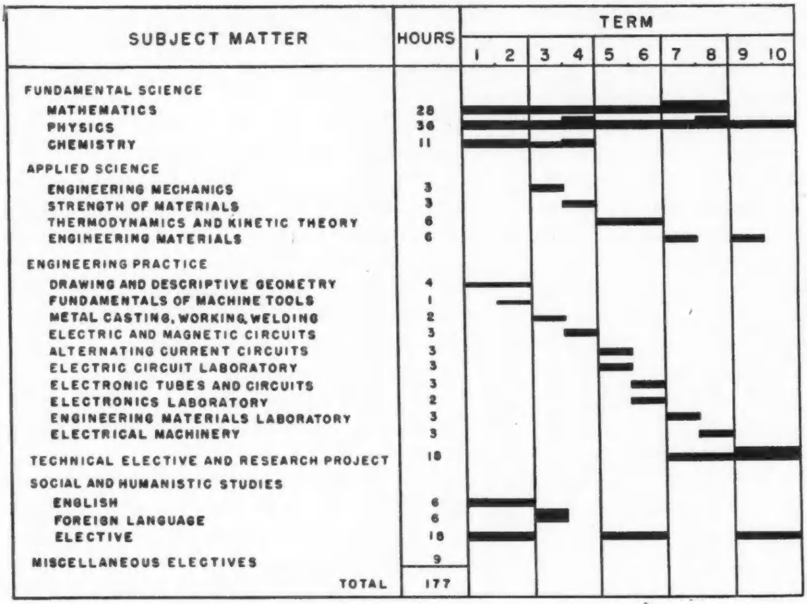


FIG. 1. Subject matter content of engineering physics curriculum together with the term in which the particular subject is given.

The actual class and laboratory instruction is spread through all pertinent departments and engineering schools. It is felt that this arrangement leads to a high degree of integration of subject matter and to a healthy cross-stimulation between basic and applied science, and facilitates the exchange of new scientific knowledge and technical methods between the basic sciences and engineering.

## THE CURRICULUM

Before we look at the details of the curriculum, there are several general observations that can be made concerning the program. As set up it requires a student to do intensive work over a five-year period. The technical subject matter falls into three main categories: fundamental science, namely, mathematics, physics, and chemistry; the physical, chemical, and metallurgical properties of materials; and engineering methods and practices. In addition there are 30 hours out of the total of 177 hours covering relevant social or humanistic studies. Of these 30 hours, 6 hours of English and 6 hours of a foreign language are required and the remaining 18 hours are elective. A considerable amount of flexibility in the technical courses is provided in the last few terms of the curriculum to allow students to advance in some technical fields beyond the level permitted by the required courses as their special interests develop in such fields. Eighteen semester hours are reserved for this purpose.

To allow the student to obtain some experience in research, he is encouraged to carry out a semi-research project in a special field of his own choice, under the supervision of a faculty member who is an authority in the special field. There are of course a great variety of such fields. They include such topics as electron physics, atomic physics, physical optics, electron optics, including electron diffraction and microscopy, x-rays and crystal structure, spectroscopy, nuclear physics, engineering electronics, communications, electrical machinery, ultra-high frequency generation and propagation, circuit analysis, elasticity and stress analysis, physical metallurgy, properties of materials, engineering mechanics, aerodynamics, etc.

In order to obtain a general view of the subject matter content of the engineering physics curriculum I have prepared the following chart, Fig. 1, which lists the subject matter under four main categories, namely, fundamental sciences, applied

science, engineering, and social or humanistic studies. The main subjects under each one of these headings are listed together with the term in which the particular subject is given within five-year course.

The general features of the curriculum are rather obvious from the chart. Some things which are not obvious though are the way in which the subject matter is integrated. As an example, it will be noted that in the fundamental sciences mathematics and physics are fairly well distributed through the entire 10 semesters and the physics is taught in such a way that the mathematics the student acquires as he proceeds is made use of in the physics courses as fast as it is acquired. This type of integration holds for a great number of other courses as well. For example, chemistry and physics are used in such engineering courses as the electrical engineering courses and the courses in engineering materials, etc. Very careful attention is paid to the fusion of the subject matter so that the program as a whole achieves a unified character.

### APPRAISAL OF THE PROGRAM TO DATE

It is, of course, impossible to make a thorough appraisal of the program as yet because of the fact that there never has been a class graduated. The first class to graduate will be in June of 1951. Consequently, we have no reports of the accomplishments from industrial laboratories concerning the people completely trained in the program. However, we have had an opportunity to make some appraisal by comparing the accomplishments of these students in their advanced work to those of students we know who are either majors in physics or pursuing other engineering courses. In addition, we have encouraged students in this program, after their third and fourth year, to obtain summer positions in industrial laboratories or in organizations in which they could put to use their technical background. We have encouraged students to take summer employment and have gone to considerable trouble to help them obtain summer employment, but we do not believe in arranging cooperative programs with industry. We feel that this leads, to a certain extent, to a rather complacent performance of the student in the cooperating industry because it is all part of his training program. We feel that it is better that the student should obtain experience as early as possible in normal employee-employer relationships so that he may appreciate the nature of his responsibilities in turning out good work and learning at first hand what the relationships with a normal employer will ultimately be. Students in the program have been employed in the summer in laboratories connected with such companies as Bell Telephone, Radio Corporation of America, Eastman Kodak, General Motors, the Oak Ridge National Laboratory, etc. In many cases the employer has written us commending the effectiveness of the student's training. In other cases we have sought reports and have uniformly found that employers have a high opinion of the work of these students and would like to have them return in the future. Consequently, with the evidence we have at hand, we feel that the educational experiment involved in the engineering physics program shows great promise of success.

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*There were, however, other crafts which did not yield so readily to the sort of treatment that transformed surveying into geometry. These were the crafts roughly corresponding to what is now the science of chemistry. They were laborious crafts, involving the use of fire, and carried on by grimy, unlettered workers, from whom the Greek thinkers were almost completely cut off by their social position. Hence the chemical theory of the Greeks was a fiasco; that was the price they paid for neglecting the rich stores of knowledge about the stuff of the world which these humble craftsmen had accumulated through so many centuries.*—A. ARMITAGE, *Sun Stand Thou Still*, H. Schuman, New York, 1946.

# A Graphical Solution for the Series Impedance Equivalent to Two Impedances in Parallel

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(Received May 16, 1950)

The paper points out a simple, but exact, graphical solution for the series impedance as a vector in the complex impedance plane, obtained geometrically from the two given impedance vectors in parallel.

THE value of the resistance and the reactance in series that would be equivalent to two impedances in parallel is of frequent interest in ac circuit calculations. These values are usually given as algebraic expressions in terms of the resistances and the reactances in the parallel branches; approximate formulas are in common use when one branch of the parallel circuit is nearly a pure resistance and the other, nearly pure reactance. We deal here with a graphical solution. It should be mentioned that a graphical solution of ac circuits in general was given in a paper by Winans, Cole, Walters, and Hummel.[1] In that paper, however, the treatment of the case of impedances in parallel was not exclusively graphical, in the way that the present paper aims at.

In the special case of a pure resistance and a pure reactance in parallel, the solution leads to the simple construction of letting a perpendicular fall upon the hypotenuse of a right-angle triangle (Fig. 3). However, even this simple construction for this special case of common occurrence, although mentioned previously in the literature,[2] does not seem to have appeared in textbooks on ac circuits and is therefore not widely known.

## GRAPHICAL CONSTRUCTION

Referring to Fig. 1, given are the two arbitrary impedance vectors $OA$ and $AB$ in a parallel circuit, drawn consecutively in the complex impedance plane. A vector $OC$ is drawn from $O$ which is the mirror reflection of the vector $AB$ upon the perpendicular bisector of $OA$. In the triangle $ACO$, if the line $OP$ is drawn from $O$ to a point $P$ on the segment $AC$, or on its extension beyond $C$, such that the angle $AOP$ equals the vertex angle $ACO$, then the vector $OP$ represents the series impedance equivalent to the two impedances $OA$ and $AB$ in parallel.

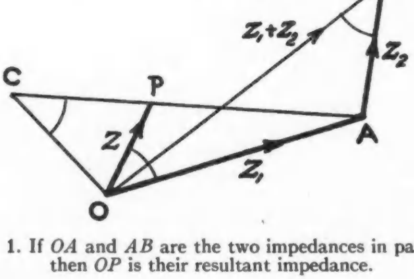


FIG. 1. If $OA$ and $AB$ are the two impedances in parallel, then $OP$ is their resultant impedance.

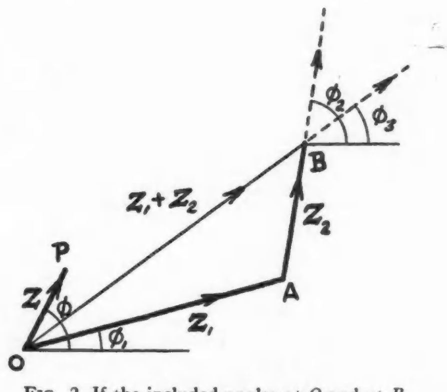


FIG. 2. If the included angles at $O$ and at $B$ are equal, then $\phi-\phi_1=\phi_2-\phi_3$.

[1] Winans, Cole, Walters, and Hummell, Am. J. Phys. 17, 503 (1949).

[2] B. Chance *et al.*, *Waveforms*, Radiation Laboratory Series, Vol. 19 (McGraw-Hill Book Company, Inc., New York, 1949), p. 139. However, a remark given in a footnote on that page, that this construction applies equally to the general case of any two impedances, fails to take account of the reflection process which appears to be an essential step in the present solution.

### PROOF

Let us first write down expressions for the several complex quantities in the problem in terms of their respective amplitudes and phase angles. Thus,

$$Z_1 = |Z_1|e^{i\phi_1}, \quad Z_2 = |Z_2|e^{i\phi_2}, \quad (Z_1+Z_2) = |Z_1+Z_2|e^{i\phi_3};$$

and

$$OP = Z = |OP|e^{i\phi}.$$

We have to show that $Z = Z_1Z_2/(Z_1+Z_2)$, or

$$|OP|e^{i\phi} = (|Z_1|\,|Z_2|/|Z_1+Z_2|)e^{i(\phi_1+\phi_2-\phi_3)};$$

this is equivalent to requiring that

$$|OP| = |Z_1|\,|Z_2|/(|Z_1+Z_2|), \tag{1}$$

and that

$$\phi = \phi_1+\phi_2-\phi_3. \tag{2}$$

1. To prove the equality (1), we observe that $|Z_1+Z_2| = |OB| = |AC|$. Now the triangles $ACO$ and $AOP$ are similar; hence

$$|OP| = |OA|\,|OC|/|AC| = |Z_1|\,|Z_2|/|Z_1+Z_2|.$$

2. To prove the equality (2), we turn to Fig. 2. We extend the vectors $AB$ and $OB$ beyond the point $B$, as shown by the dotted arrows (their amplitudes, being irrelevant in this part of the proof, are not shown to scale). Now, considering the pair of vectors at the point $O$, the difference of their phase angles, $\phi-\phi_1$, is equal to the magnitude of their included angle. Similarly, the included angle at point $B$ is equal to $\phi_2-\phi_3$. Since, by construction, the included angles at $O$ and at $B$ are equal, we have

$$\phi-\phi_1 = \phi_2-\phi_3, \tag{3}$$

and the equality (2) is proved.

It is to be noted that whenever the vector $AB$ lies in the left half plane with reference to the vector $OA$, as is the case indicated in Fig. 1, the expressions on both sides of Eq. (3) are positive and equal. A moment's reflection will reveal that these expressions are both negative and also equal if the vector $AB$ should lie in the right half plane with reference to $OA$.

### ALTERNATE PROOF

The two steps of the foregoing proof can be combined into one by means of an extension of the geometric law of similar triangles.

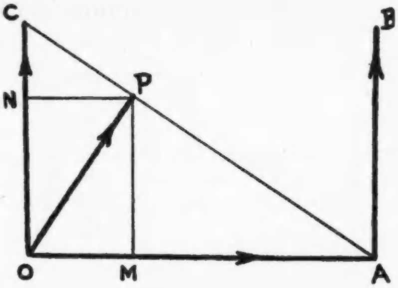


FIG. 3. If a resistance $OA$ and a reactance $OC$ are in parallel, then their resultant impedance is $OP$, the perpendicular upon the hypotenuse $AC$.

Referring again to Fig. 1, we compare the similar triangles $APO$ and $AOC$; but, aside from a change in scale, they are mirror reflections of each other.

Since, by construction, the triangle $OAB$ is a reflection of the triangle $AOC$, it follows that the triangles $OAB$ and $APO$ are similar and also *maintain the same sense.* With two such similar triangles, if the direction and the magnitude of each side are represented by a complex number, a little thought will convince the reader that the law in plane geometry regarding the proportionality of any two corresponding sides can be extended at once to the equality of the quotients of the complex numbers representing the corresponding sides.

Thus, in the present case, we have,

$$OA = Z_1, \quad AB = Z_2, \quad OB = (Z_1+Z_2), \text{ and } OP = Z.$$

From the similar triangles, we have $AB/OB = PO/AO = OP/OA$, or, $Z/Z_1 = Z_2/(Z_1+Z_2)$, i.e., $Z = Z_1Z_2/(Z_1+Z_2)$.

### SPECIAL CASE—PURE RESISTANCE AND PURE REACTANCE IN PARALLEL

In the special case of a pure resistance and a pure reactance in parallel, the above construction reduces to a much simpler rule. As in Fig. 3, the pure resistance is the vector $OA$, while the reflection $OC$ of the pure reactance vector $AB$ is simply $AB$ itself, but drawn from the origin $O$. The triangle $ACO$ being a right-angle triangle, the point $P$ is thus the foot of the perpendicular from the origin $O$ to the hypotenuse $AC$. The equivalent series resistance and reactance are then given by $OM$ and $ON$, respectively.

# A Simple Geometrical Proof of Buckingham's $\pi$-Theorem

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(Received November 3, 1950)

By introduction of the concept of a "dimension space," whose coordinates are the exponents of the basic dimensions of physical quantities, the proof of Buckingham $\pi$-theorem is transformed into a simple geometrical problem.

BUCKINGHAM'S famous $\pi$-theorem[1] may be stated as follows: *If there exists a unique relation*

$$F(A_1, A_2, \cdots, A_n) = 0$$

*among $n$ physical quantities which involve $k$ physical dimensions, then there also exists a relation*

$$\Phi(\pi_1, \pi_2, \cdots \pi_{n-k}) = 0$$

*among $(n-k)$ dimensionless products made up of the $A$'s.*

This is ordinarily proved in *a posteriori* fashion, by assuming the existence of the dimensionless $\pi$-products and deducing sets of simultaneous equations among the various pertinent exponents.[2]

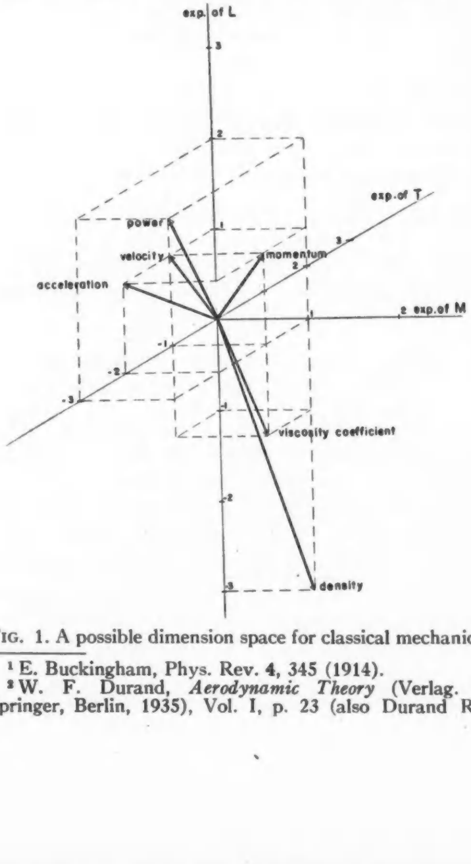


FIG. 1. A possible dimension space for classical mechanics.

Bridgman[3] has given a more general proof, which involves no explicit assumption on the form of $F$ except that it be a mathematically well-behaved function. However, his proof is also rather involved, and it would seem helpful to be able to demonstrate this theorem more briefly. Such a demonstration can be easily made with the concept of a "dimension space."

## THE DIMENSION SPACE

We define a $k$-dimensional cartesian space[4] whose coordinate axes are the exponents of the $k$ physical dimensions chosen as basic to the physical problem. For example, in problems in classical mechanics, they may possibly be exponent of M, exponent of L, and exponent of T, where M = mass, L = length, and T = time. In this space, each physical quantity is a vector which can be drawn from the origin to a point appropriate to its dimensions. Figure 1 shows a possible dimension space for classical mechanics with several common physical quantities indicated.

Dimensional equality of two sets of products of physical quantities means simply that the two corresponding vector additions in the dimension space terminate at the same point. A dimensionless product is a closed vector polygon in this $k$-dimensional "$D$-space."

## PROOF OF THE $\pi$-THEOREM

Our problem involves $n$ physical quantities $(A_1, A_2, \cdots A_n)$ in a branch of physics for which it is convenient to work with $k$ basic dimensions. Just as in the previous proofs, we choose $k$ primary quantities from the group of $n$. The requirements on these primary quantities are twofold:

[1] E. Buckingham, Phys. Rev. **4**, 345 (1914).

[2] W. F. Durand, *Aerodynamic Theory* (Verlag. J. Springer, Berlin, 1935), Vol. I, p. 23 (also Durand Reprinting Committee, California Institute of Technology Press, Pasadena, 1943).

[3] P. W. Bridgman, *Dimensional Analysis* (Yale University Press, New Haven, Connecticut, 1931), rev. edition.

[4] Nonorthogonal system would serve as well; the Cartesian assumption is merely for conceptual simplicity.

(a) They must be independent, i.e., non coincident vectors in the $D$-space; and (b) they must *not* be confined to a $(k-1)$-dimensional hyperplane in the $k$-dimensional $D$-space.*
The remaining $(n-k)$ $A$'s are termed secondary.

*Lemma: The dimensions of any one of the secondary physical quantities can be duplicated by a unique product of the primary quantities.* In the $D$-space the "primary vectors" may be considered as a new $k$-dimensional coordinate system, and then the lemma statement becomes the following: Any vector in a $k$-dimensional space is uniquely determined by its components on a $k$-dimensional coordinate system. This statement is obvious; hence, the lemma is proved.

Since there are $(n-k)$ secondary quantities, it follows that exactly $(n-k)$ dimensionless $\pi$-products can be formed by dividing each secondary quantity by the equidimensional product of primary quantities.

With no loss in generality from the point of view of dimensional analysis, we may consider $F(A_1, A_2, \cdots A_n)$ to be a dimensionally homogeneous polynomial with all transcendental functions of the $A$'s lumped into the (dimensionless) coefficients. A typical term in the polynomial would look like

$$B \cdot \underbrace{A_1^{a_1} \cdot A_2^{a_2} \cdot \ \cdots \ \cdot A_k^{a_k}}_{\text{Primary}} \cdot \underbrace{A_{k+1}^{a_{k+1}} \cdot \ \cdots \ \cdot A_n^{a_n}}_{\text{Secondary}},$$

where $B$ is a dimensionless function of the $A$'s and does not enter the discussion.

If we divide each secondary $A$ by the proper equidimensional product of primary $A$'s and multiply by the same thing, our typical term becomes

$$\beta \cdot A_1^{b_1} \cdot A_2^{b_2} \cdot \ \cdots \ \cdot A_k^{b_k},$$

where $\beta$ is dimensionless.

Because of dimensional homogeneity,[5] each of these product terms, made up only of primary quantities, has the same dimensions as every other one in the polynomial. In the $D$-space, they are all the same vector, and the exponents of the factors are the (necessarily unique) components of this vector in the coordinate system made up of the primary quantity vectors. Hence, these primary quantity products in all terms of the polynomial are mathematically identical, and can be canceled out of the equation, $F=0$, leaving an equation in terms of the dimensionless products[6] $\Phi=0$. Q.E.D.

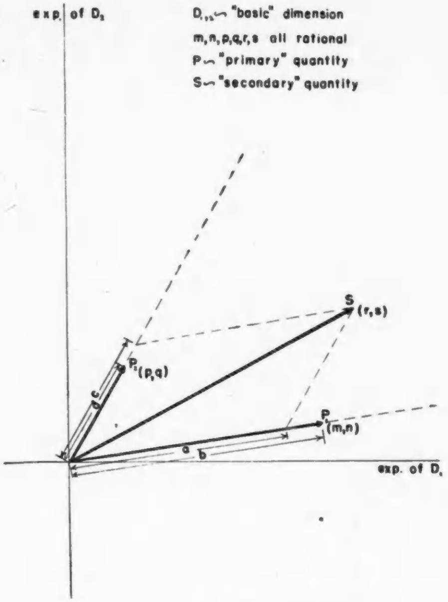


FIG. 2. A two-dimensional $D$-space.

### A COROLLARY

It can be proved that in the process of duplicating the dimensions of any secondary quantity by a product of primary quantities (lemma), the latter need be raised only to rational powers. This is virtually obvious from the initial fact that the dimensional expression for any physical quantity involves only rational powers of the basic dimensions. However, in the $D$-space it can be given a formal geometrical or a trigonometric proof. In a two-dimensional $D$-space, for example Fig. 2, we prove that if $m$, $n$, $p$, $q$, $r$, $s$ are rational, then $(a/b)$ and $(c/d)$ are also rational. Professor A. Wintner has pointed out to me that this can undoubtedly also be proved from the properties of non-unimodular lattice transformations in the theory of numbers.

I should like to acknowledge the helpful criticism of Dr. Leslie S. G. Kovasznay and Mr. A. Kistler.

* Note added in proof: Since (b) implies (a), it is the only requirement.

[5] Found to exist in all constructively useful physical equations.

[6] The dimensionless arguments of the transcendental functions can always be expressed in terms of the $\pi$-products.

# The Teaching of Physics*

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(Received November 16, 1950)

Observations upon the teaching of physics, with special reference, in the case of courses intended for cultural purposes, to the importance of emphasis on ideas rather than facts, except insofar as they are necessary to illustrate the ideas.

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THE teaching of physics, as indeed the teaching of anything else in school or college, is apt to become modified with the lapse of time in such manner that its primary functions are diluted in some measure by the necessities of administration. The effect of such dilution is to be appraised through an examination of the end product. Insofar as it may be maintained that dilution is necessary and that there is no escape from its evils, it is well to examine the results of teaching in different localities—in different nations, preferably—where these processes have operated, perhaps in different forms and in different degrees.

It is, I believe, generally recognized that the young European student of the pre-undergraduate stage is ahead of his American contemporary by perhaps two or three years, both in the maturity of his knowledge and in the amount of it; in this comparison I would certainly place maturity ahead of amount.

It is well to think of physics as comprising two phases, knowledge of facts and knowledge of ideas. By knowledge of ideas I mean the kind of mental adaptation which makes the student a critical appraiser of that which he learns. In certain other branches of knowledge, in the social sciences, for example, and even in the arts, the junior student has frequently strong convictions concerning what he learns. He may find himself wrong in those convictions, but I believe it is better to have wrong convictions than no convictions at all.

Now those things which, in physics, represent the territory of strong convictions are, in the social sciences, matters calling frequently for a more difficult mental effort than is called for in the nonscientific fields; and there is danger that in the scientific fields the student and teacher may take the easy path of relieving themselves from mental exertion by learning or teaching as facts, without the urge for correlation or appraisal, those things which are essentially of the realm of ideas. There is danger that the student will learn that for a falling body $\frac{1}{2}mv^2=mgh$ as a starting point, calling for no further meditation than is called for in his knowledge of the fact that water is a liquid and copper a solid at ordinary temperatures.

I think we have to envisage two different purposes in the teaching of physics. If the student is planning to be what I may call a technician—perhaps the word is too specialized for the purpose I have in mind, but it will serve in lieu of something better—if he is planning to be a technician, then I think it is safe to say that the facts assume for him a primary importance, and he needs the ideas only to the extent necessary for the manipulation of the facts. It will be very useful for him to know of all of the different kinds of radio tubes which exist and of their characteristics. It will be useful for him to know what condensers leak and what condensers are well insulated. But if the student is studying physics as a mental discipline, then I will venture the heresy that ideas are all important and that facts are necessary only to the extent sufficient to provide material for the manipulation of the ideas.

Educated people of the nonspecialist class are concerned with science largely in relation to its ideas. The man in the street is intrigued by what he hears on the theory of relativity. He is much interested in new thoughts about the origin of

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* Based upon an address delivered at the Pennsylvania Conference of College Physics Teachers at Haverford College, October 20, 1950 and at the twentieth annual meeting of the AAPT at Barnard College and Columbia University, New York, February 3, 1951.

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the universe. He is much interested in what he hears about nuclear physics, the properties of atomic particles, and the atomic bomb. And yet, alas, is it not true that after a year of physics in high school and a few more in the earlier stages of university education, and with high qualifications in the domain of his specialized calling, we find him, later, a successful man, perhaps a dean of a college or even a president of a university, and yet in the state of mind in which he is unable to discuss a scientific article in the *New York Times* with a degree of sophistication equal to that with which two old ladies will discuss a new book at a tea party, or with which two road diggers will discuss the political situation. It is true that the road diggers may discuss the political situation in a highly illogical manner, but at least they will have something which they can use to argue back and forth, something which at least has a chance of being moulded into something else which *is* logical.

Now, where lies the difficulty in this matter? Of course, it is natural to say that physics is a very specialized subject calling for a certain type of mentality which is the possession of only a few; but while this may in part be true, I do not believe that it is true to the extent necessary to explain the mental calamity to which I have referred. May it not be that much of the difficulty lies in the fact that we have fled from the realm of ideas to seek what appears to be a safer haven in the territory of facts?

I believe that perhaps one of the most potent influences tending to the development of mediocrity in thought is to be found in the necessity of testing the progress of the student as he learns, in the examination system, for example. If it is necessary every few weeks to set a group of half a dozen questions to test what the student has acquired, it is much easier to have questions which permit an answer in terms of facts, or in a standardized system of words invented to describe principles, than it is to set questions which necessitate answers which come from the brain rather than from the memory. It is convenient for the examiner if the answers are all more or less alike in method and wording.

I remember once as a young student being asked in a test question to state the "law of multiple proportions." Now I knew what the law of multiple proportions was, but I could never remember the official jargon in which it was expressed, so I expressed it in my own way. The young instructor said: "If Mr. Jones were marking this paper, he would give absolutely no marks for that answer" (Mr. Jones was the head of the course). "But why," I said, "is it wrong?" "No," said he, "it is right, but you have not expressed it in the usual way." I could not help remarking: "Well then, I think that would be very unjust of Mr. Jones."

Of course, I will admit that it might be a terrible nuisance to an instructor to have a couple of dozen students expressing the "law of multiple proportions" in different ways in an examination, even though they might all be right; for naturally, the poor instructor would have to twist his brain into all sorts of contortions to verify the correctness of the various answers. And yet, a parrot-like learning of stereotyped phrases is apt to produce calamitous results, as was the case with what I once read in an examination paper as the definition of a "dyne": "A dyne is that force which, when placed one centimeter away from a magnetic pole of exactly similar strength, repels it with the force of one dyne."

When I was professor at one of our greater universities, a student came to me and said: "Professor, are we supposed to memorize all of your lectures?" I fear this question drove me almost to the realm of profanity, for I replied: "My God, man, such a thing is impossible. Wherever did you get that idea?" "Well," he said, "I fear that unless I do memorize them I shall be unable to pass the examination, because you are using the calculus of variations and I have never had a course in the calculus of variations." "Fine," said I, "neither have I had a course in the calculus of variations. Moreover, I have never had a course on the subject (electrodynamics) I am lecturing to you about now." I fear the poor fellow was completely flabbergasted, but in time he saw the light and became a very faithful disciple.

Many of the subjects one learns in schools and elsewhere, subjects like geography, history, languages, and even anatomy, are matters involving knowledge of facts to a much greater extent than are the physical sciences. Many of these sub-

jects were fields of learning before science was studied, and so there grew up a cult for the glorification of facts. He who knew many facts was a learned man. He who had only ideas was a dreamer, a lazy fellow, who never might be expected to make his way in the world. One sees a remnant of this glorification of mere knowledge in the long words which adorn some of the sciences which got under way before physics became recognized by any but the select few. The medical and allied sciences are loaded with huge names, which, if prattled off with sufficient assurance, can produce a marvelous impression on those who do not know what they mean. Even chemistry is not without its very long names, although here the science has become well leavened with ideas. Chemistry, by its very nature, cannot escape the burden of innumerable different facts and situations.

Now, of course, I am not criticizing the chemist for his innumerable theorems. A set of self-consistent fundamental principles can indeed be thrown into all sorts of different forms, and while one form is very suitable for a certain class of problem, another form is more suitable for another class of problem. In fact, it is part of the art of the theorist to throw his theorems into forms which are most adaptable to the immediate purpose which they have to serve. The law of inverse squares, as expressed in terms of the forces between particles, seems, in this expression, to assume its most simple form. And yet, in this form it taxed the brain of the great Newton to show that a sphere produced at a distance a force which was the same as if all its matter had been concentrated at its center, while in the form of Gauss' theorem, which is the analytical equivalent of the law of inverse squares, the thing can be proved in a space no larger than is provided by the back of a postage stamp.

And so we must recognize that the expression of theorems in different forms is a thing of great value. However, it should be the purpose of the teacher to point out to the student, very clearly, the interrelation of these forms, so that the mind has a secure anchorage to which they are all bound.

I think that perhaps in elementary courses a great many things are learned which are quite unnecessary. I remember that, in my young days at any rate, all textbooks on physics had a section devoted to pointing out that water had a maximum density at 4 degrees centigrade. There was an apparatus called Hope's apparatus, with two horizontal thermometers in it, one above the other, and intended to show that under suitable conditions the lower thermometer registered higher than the upper one. The importance given to this picture naturally tended to convey to the student the idea that this experiment was one of the fundamental landmarks of science and that Hope was one of the great pioneers thereof. Any student who had the audacity to think for himself would realize that any fool could invent an apparatus like that and would wonder what there was in the rest of the science, if so much was to be made of this matter. As a matter of fact, I think it is quite unimportant to know that water has a point of maximum density, and the same thing may be said of a lot of other facts which occasionally encumber the mind of the student.

Then as regards lecture experiments, many such experiments seek to illustrate principles in a form which represents them in very complicated guise, the reason being that the experiment provides a very good show in this form. Such experiments may be very entertaining, and they can provide in their description easy material to call for in an examination, but I doubt whether they contribute anything to the mental advancement of the student.

I believe we could dispense with a great deal of material of the factual kind and provide much more time for a clear understanding of what is involved in some of the fundamental principles of our science—what is involved, for example, in the so-called laws of dynamics—what is involved in the motion of a planet around the sun—how it comes about that we talk in this language, anyway.

Of course, much of what I have said applies to teaching in the elementary stage, although to me, at any rate, advanced physics is, in a large part, a more thorough understanding of the principles of elementary physics. Physics is largely an attitude of mind and I like to think that if I should go to bed tonight and wake up in the morning to find that I had forgotten everything that I had ever learned, but had succeeded in retaining such experience as I have in thinking, I should not have suffered very much by the

loss. It would, of course, be a little inconvenient to fail to have ready at hand some of the formulas and methods which are so familiar to us, but this loss could soon be repaired.

It usually happens that the student who is ultimately going to be a professional physicist eventually flounders into realms of logical thinking, however much he may have been impeded by his earlier education. And at this stage the difference between the educational standards here and abroad vanishes, and those who have made the subject their own, stand on a plane of equality in logical thinking in all races and in all countries. However, it is a pity that the other people who have not developed to this happy state should be left in the unfortunate condition in which, undoubtedly, most of them are to be found. Part of the fault lies, possibly, with the formulation of standardized systems of education in which emphasis is frequently made in the wrong place. It is easy to become intrigued with the idea of persuading the student to observe all that is around him. The trait characterized by inquisitive observations was, at an early stage, put forth as something to be worshipped with awe by all students, and the genus "inventor" was idolized. I have no brief against observation. Once one is launched upon some definite endeavor in science, careful observation is necessary to define the essential elements which should claim attention. However, this is a different matter from allowing oneself to become intrigued by the workings of the automobile, the airplane, the steam engine, the radio, the television, and so forth, and beguiling oneself into the belief that one is educating a student by providing him with the superficial jargon in terms of which half-understood principles, or less than half-understood principles, are exploited, diluted with conglomerations of gears and mechanisms, in such manner as to have little chance of germinating in his mind into anything more than the conglomeration of misunderstood statements. I will confess that I see no solution of this matter other than one which has the courage to omit an attempt to make what is taught have immediate application over the whole realm of scientific affairs, and concentrate rather upon the fundamental ideas in a form which, at the same time, is more elementary but also more profound. A thorough understanding of elementary principles leaves but a small step to an understanding of the essentials of their practical application, but an attempt to reach the practical application by too superficial a route must inevitably result in the student failing to understand both the principles and the application.

It is unfortunately a fact in physics, and indeed in other things, like music, that it is at the beginning of his studies that the student needs the most mature minds to guide him. This is the period in which his standards of critical judgment are being formed; but alas, it is at this stage that he is usually provided with a teacher whose own training and mental discipline have never taken him beyond the superficialities of thought.

Even in our universities, the so-called successful teacher is frequently one who, either through lack of urge in the matter or through endeavor to produce so-called concrete results, has sacrificed the ideal to the seemingly practical. He is frequently one who possesses the power to systematize his courses so that the student is left with no uncertainty as to what he is supposed to learn and what he is supposed to do. Everything is neatly laid out and the student works hard and passes his tests frequently, alas, with very little comprehension of what he has been doing.

As I have grown older, I have come more and more to the conclusion that there is no teaching in physics, there is only inspiration to learn. Most of us are familiar with the situation in which, in meeting some new idea or demonstration of the consequences of an idea, we understand, in the sense that we can reproduce the arguments, and indeed can follow them step by step as presented, but in which we are left without that deep-seated conviction of understanding which is the only thing which can safely form a basis for use of the information we have gained. Then, perhaps six months later and possibly without conscious meditation in the time between, a light suddenly dawns on the matter and all seems clear. I believe that until this has happened, nothing of value has happened. The teacher may stimulate the mind of the student to reach this goal, but the journey to that goal must be made by the student himself. And so again I say that there is no teaching—there is only inspiration to learn; and it

may not be much of an exaggeration to maintain that if the teacher ever did succeed in teaching anything to enable the student to avoid the necessity of making his own journey, he would, by that very act, have killed for all time the possibilities of the poor student reaching the goal which he sought. Instead of being inside the citadel to be found at the goal and eagerly conscious of what was around, he would be left standing outside, enjoying only a crooked vision of what was going on inside, a vision such as might be obtained through a window marred with distorting features. In seeking to understand new ideas, the student must, in a sense, travel the same path as the originator of the ideas. To do this, however, he does not have to be a Newton or an Einstein, for he has beside him his teacher to steer him away from unfruitful paths and illuminate the beauties of the true path as he develops eyes to see it.

I have appeared to speak disparagingly of certain aspects of examinations. However, I do not wish to imply that examinations are unnecessary. Neither do I think that a system of overspecialization should bring the physicist to a state where he knows everything about one thing and nothing about anything else. I believe that every physicist should know how to determine the focal length of a thin lens and the nodal points of a thick one. I think that every physicist should, in fact, be acquainted with the fundamental ideas and phenomena illustrating them which pertain to the whole range of our science. It may be that some of these matters will go into shadow as he specializes on nuclear physics or electronics; but there should be some time in his student career in which he has them all assembled in his mind more or less simultaneously, not perhaps with the sophistication in every field which would be demanded from specialists in the fields, but at any rate with the sophistication which implies a practical ability to use the principles. If in future years some of those matters become partially forgotten, they can easily be recalled and put into service when needed. I think, therefore, that the principle of the comprehensive written examination extending over a long period, perhaps several days, is a good one.

As regards the examination in practical physics, I could say much. I believe that examinations which ask the student to measure the given induction or find the specific heat of a given solid do not test very much. Once, many years ago, when I had an occasion to set an examination for fairly advanced students, I adopted a novel principle. Instead of setting, for each student, one thing to measure, about a dozen experiments were all set up, more or less correctly. However, in every one of them there were one or two things which were wrong. Perhaps this apparatus had a leaky condenser. In another apparatus there was a break in a wire. In another the cotton surrounding the wire was gripped in the galvanometer terminal. In another the insulation of one part of the apparatus was not good enough to enable the resistance of another part to be measured. It was the business of the student to go to each experiment, find what was wrong, and put it right.

I sometimes regret that much of our modern apparatus, even for students, has all the interesting difficulties removed beforehand. If a student is going to work with a spectrometer, I think it is highly desirable that he should go through the process of adjusting the collimator, the telescope, and eye piece himself. It is desirable that he shall go through the process of getting the grating lines parallel with the axis of rotation. It is desirable that he shall know how to set the axis of the telescope perpendicular to the axis of rotation. Once the spectrometer is adjusted, all of the good of the experiment has been utilized. I do not think that the student learns much in the last act of measuring the wavelength of light.

Now, of course, it is a highly immoral thing to cook the results of experiments and, naturally, I do not advocate it, and yet—but tell it not in Gath—some valuable training is involved in cooking the experiment. The student gets a certain result which doesn't look right. He finds he can make it better by falsifying the length of the wire; but this does not do very much because the wire comes in only as the first power. He tries some other parameter, but this is involved only in its logarithm and not much is to be gained by fooling with that. On the other hand, in another part of the expression, the fourth power of the radius of the wire is involved. He will gain much by doing a little cook-

ing on the radius of the wire. Now, while all of this is highly immoral, the kind of thinking which our wicked student is doing in falsifying his results is exactly the kind of thinking which he should do when working in the capacity of an honest experimentalist.

Much can be said about oral examinations for doctor's degrees and in my judgment not much can be said that is good. I have sat in innumerable examinations for Ph.D. at very many different universities, sometimes as a member of the permanent faculty and sometimes as a visitor. In almost every case the knowledge exhibited was such that if it represented the true state of mind of the student, he never should have been passed. However, after the examination is concluded, there is usually a discussion to the effect that: "Well, So-and-so got tied up pretty badly, but I happen to know that he is a very good man," etc., etc., and so finally he is passed.

Frequently the question involves simply reproducing some standard piece of material which happens to be dear to the heart of the examiner who asks the question. Then, in nine cases out of ten, the student drops a 2 somewhere or gets a $\pi$ in the wrong place. Of course, this error has no significance at all in relation to the understanding of the subject, but frequently the poor devil is tormented to distraction in an endeavor to make him find the 2 or the $\pi$. He gets more and more befuddled and soon is in a state of mind in which nobody in that state of mind, the examiner not excluded, could find the 2 or the $\pi$ or anything else.

And yet, strange to say, in spite of the evils of these stereotyped questions, such questions have been so ingrained in the mind of the candidate that he is disturbed if one makes any other approach. Before the examination, he has probably tried to size up the personalities of the various examiners, their likes and dislikes, and make a guess at the questions they are likely to ask. The measure of the candidate's happiness is his success in the guesses.

Now frequently I have asked a question which seems the most reasonable of all questions and yet which, for some reason or other, seems to be regarded as most unfair because the question departs from the usual category. The question is as follows: "Mr. X, is there any particular branch of physics in which you are specially interested?" Possibly Mr. X says that he has given special attention to thermodynamics. Possibly he finds that there is nothing that he can cite in which he is interested. In the former case, the procedure is definite and in the latter case a subject has to be suggested for him. Taking the former case, I have then said: "Now, Mr. X, I should like you to suppose that I am an intelligent person, familiar with the general methods of thinking in science, but that I have never heard of thermodynamics. I should like you to talk to me for about fifteen minutes and try to give me an idea of what thermodynamics is about, what its uses are, and so forth. I make only one stipulation and that is that I shall discuss the matter with you as you proceed." Alas, as I have said, such a question is usually regarded as a most cruel imposition on the mental capacities of the student.

I think we are all acquainted with the situation in which, having been introduced to Mr. So-and-so who is said to be a physicist and whom we have not met before, we sit down by the fire and start to talk. If the discussion is led into the realm where our visitor starts to talk about his subject, we all know that before half an hour has passed we have a much more complete concept of the ability of our guest as a physicist than we could have gained from a three hours Ph.D. examination of the usual type. And so I have wondered whether the oral Ph.D. examination could not be replaced by what I may call a series of conversations in which, from time to time, the various examiners, without preannouncement, and indeed in the spirit of concealing as far as possible that they are doing anything official, engage in some meaningful conversation with the candidate. If notes of the results of such conversations are made, the summation of all of these notes can very well provide, in my judgment, a more meaningful examination than the usual formal oral Ph.D. examination.

And now I must conclude. If anything I have said brings a chord of response in any of my audience, I am happy. If I have trodden on any toes, I apologize. At the worst, if I am all wrong I am material for conversion, and I am happy in the thought that there is more joy in Heaven over one sinner that repenteth than over the ninety and nine who need no repentance.

# NOTES AND DISCUSSION

## An Experiment Illustrating the Elliptic Integral of the First Kind

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WE have found the following experiment useful in introducing the concept of the elliptic function as it occurs in the problem of the large amplitude oscillations of a simple pendulum. Such an experiment has been performed in the intermediate mechanics laboratory at the University of Pittsburgh with very good results.

If one considers the oscillations of a pendulum of length $L$, oscillating with a maximum amplitude, $\theta_0$, the period $T$ of the oscillation is given by

$$T=4(L/g)^{\frac{1}{2}}\int_0^{\pi/2}\frac{d\phi}{(1-\sin^2\theta_0/2\,\sin^2\phi)^{\frac{1}{2}}},\tag{1}$$

where $\sin\theta/2=\sin\theta_0/2\,\sin\phi$, and $\theta$ is the angle between the vertical and the pendulum.

The complete elliptic integral of the first kind is defined by

$$K(\sin\theta_0/2)=\int_0^{\pi/2}\frac{d\phi}{(1-\sin^2\theta_0/2\,\sin^2\phi)^{\frac{1}{2}}}.$$

Therefore,

$$T=4(L/g)^{\frac{1}{2}}K(\sin\theta_0/2).\tag{2}$$

For small values of $\theta$, $\theta_0\cong\sin\theta_0$, and, $K\rightarrow\pi/2$. In this case the period $T$ assumes the well-known form, $2\pi(L/g)^{\frac{1}{2}}$.

Consider now, two simple pendulums of lengths $L_1$ and $L_2$, respectively. Adjust $L_1$ to equal $L_2$; this causes the fundamental periods of the two pendulums to be equal. If pendulum number 1 is allowed to oscillate with a large amplitude $\theta_0$, the period $T_1$ is given by Eq. (2)

$$T_1=4(L_1/g)^{\frac{1}{2}}K(\sin\theta_0/2).\tag{3}$$

If pendulum number 2 oscillates with a small amplitude ($\leq 5°$), the period $T_2$ is given by

$$T_2=2\pi(L_2/g)^{\frac{1}{2}}.\tag{4}$$

Obviously, the complete integral of the first kind, $K\sin(\theta_0/2)$, can be computed from the direct ratio of $T_1$ and $T_2$. However, in such a calculation, measurements of $T$ must be carried out with an accuracy comparable to the accuracy desired for the value of $K$. The integral $K(\sin\theta_0/2)$ is equal to $\pi/2$ in the limit as $\theta_0\rightarrow 0°$, and at $\theta_0=90°$ is only greater than $\pi/2$ by 18 percent. By measuring the beat frequency between pendulum number 1 and pendulum number 2 one can compute the difference between $K$ and $\pi/2$ with an equivalent accuracy.

Take the ratio of Eq. (4) to Eq. (3), considering $L_1=L_2$. Then,

$$\frac{1}{K(\sin\theta_0/2)}=\frac{2T_2}{\pi T_1}\cong\frac{2f_1}{\pi f_2}.\tag{5}$$

Subtract $2/\pi$ from each side of Eq. (5) and $K$ can be obtained in the convenient form,

$$K(\sin\theta_0/2)=\frac{\pi/2}{(1-\Delta f T_2)}.\tag{6}$$

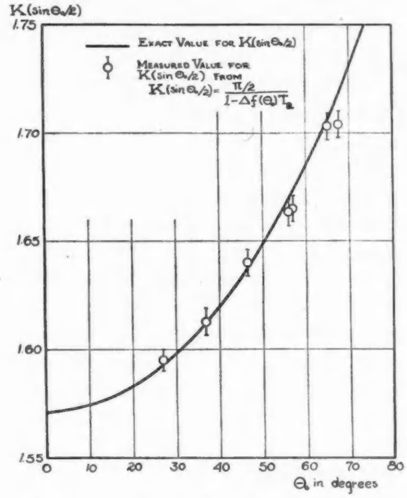


FIG. 1. Plot of the elliptic integral of the first kind $K(\sin\theta_0/2)$. The full line represents the theoretical values; the individually plotted points are derived from observations on equal pendulums oscillating with different amplitudes.

Here, $\Delta f$ is the beat frequency between pendulum 1 and pendulum 2. The product $\Delta f T_2$ is small compared to unity for angles up to 90°. Therefore, an experimental variation in $\Delta f T_2$ will cause a much smaller variation in the value for $K$ as calculated from Eq. (6).

If the lengths of the two pendulums are not exactly equal, a small correction factor must be added to Eq. (6);

$$K(\sin\theta_0/2)=\frac{\pi/2}{(1-\Delta f T_2)}\left[\frac{L_2}{L_1}\right]^{1/2}.\tag{7}$$

Figure 1 presents the results of a typical experiment. The beat frequency was measured by timing one phase interchange.

There are two major inaccuracies in this experiment. First, it is difficult to record the time for one phase interchange to better than about seven percent. By taking each reading a large number of times and by allowing each of two observers to take half the readings, the accuracy in timing can be increased to about one percent. Second, air damping of the oscillations throughout a run decreases the amplitude. However, if one can obtain the initial and final amplitude of the pendulum for each run, the average value can be computed. A large photoflood lamp can be rigidly supported over the setup, and the shadows, cast on a large sheet of graph paper by pendulum number 1 held at various angles, can be marked and utilized as an amplitude detector.

This experiment has the advantage of illustrating an advanced mathematical concept coupled with the use of the simplest of apparatus. We should like to thank Professor W. C. Kelly for his helpful suggestions and comments.

# LETTERS TO THE EDITOR

## Which is the More Accurate?

THE conditions that determine relative humidity are various and variable. It is no wonder, therefore, that the student of first-year college physics obtains such inconsistent results when performing the experiment on measurement of relative humidity by the dew point hygrometer and sling psychrometer methods and comparing results. He is further mystified when asked, "Which of the two methods is the more accurate?"

Upon referring to several reliable authorities he finds that "measuring the temperature of the dew point is the most accurate method of determining relative humidity."[1] Several other authors are in agreement with this statement.[2] Another textbook states that "high precision is seldom necessary or possible in measuring relative humidity."[3] Millikan, on the other hand, states that the dew point can be measured to 0.1°C "with practice."[4] With further research the student finds another reliable authority who, states, "In practice the method just described (dew point) for finding atmospheric humidity is neither convenient nor accurate."[5]

Undoubtedly each of the authors is correct in his interpretation of the situation; but the student finds himself in somewhat of a dilemma in his attempt to determine which of the methods is better. Further elaboration is needed in the standard references. It is agreed that the dew point method, when performed under controlled laboratory conditions and with proper technique, is the more accurate. Such conditions are virtually impossible to obtain when working in a group laboratory. The student should therefore be told that he should expect more reliable and valid results with the simpler but less accurate method, the sling psychrometer and a set of tables, and that the dew point hygrometer method is capable of yielding more accurate results under conditions which he is not able to obtain without taking great pains and time to achieve the proper laboratory technique (i.e., not breathing on the hygrometer, etc.). Is there a better way to explain it to him?

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[1] Sears, *Principles of Physics* (Addison-Wesley Press, Inc., Cambridge, 1950), Vol. 1, p. 317.
[2] Shortley and Williams, *Physics* (Prentice-Hall, Inc., New York, 1950), Vol. 1, p. 421.
[3] Mendenhall, Eve, Keys, and Sutton, *College Physics* (D. C. Heath and Company, Boston, 1950), p. 274.
[4] Millikan, *Mechanics, Molecular Physics, and Heat* (Ginn and Company, New York, 1903), p. 168.
[5] Margenau, Watson, and Montgomery, *Physics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 316.

## Energy Independent of Mass in Simple Harmonic Motion

A BODY of mass $m$ is attached to one end of a helical spring resting on a smooth horizontal table top. The other end of the spring is fixed in place. Displacement of the mass from the equilibrium position, in either direction along the axis of the spring, followed by its release will result in simple harmonic motion. Since there are no changes in gravitational potential energy, it is easy to see that all changes in the potential energy of the vibrating system at any instant must occur in the spring alone. The instantaneous value of the potential energy in the system is $\frac{1}{2}kx^2$, where $k=-F/x$ is the force constant of the spring, and $x$ the instantaneous displacement. The maximum value is $\frac{1}{2}kA^2$, where $A$ is amplitude. At any instant this must also equal the total energy, both potential and kinetic, for the system is regarded as conservative.

Since $k$ is a constant of the spring alone, the potential energy is independent of the mass. This is untrue, however, of the vibrational *frequency* and its reciprocal the *period*. Not only the potential energy but also the kinetic energy is at any instant independent of the mass. When this is pointed out to the beginning student, it may appear to him paradoxical. He has learned that kinetic energy $=\frac{1}{2}mv^2$, which contains mass, but as can be shown for the present case, $v^2=4\pi^2n^2(A^2-x^2)$, where $n$ is the frequency; and since $n^2=\omega^2/(4\pi^2)=k/(4\pi^2m)$, we have $v^2=k(A^2-x^2)/m$, so that $\frac{1}{2}mv^2=k(A^2-x^2)/2$, which does not contain the mass.

To give the student a physical picture, it can be pointed out that *using the same spring*, and the same amplitude, which were assumed above, a heavier mass slows down the frequency just enough so that the product $\frac{1}{2}mv^2$ remains the same for any displacement $x$, that is, for any particular fixed point in the path of the vibrating body. This is simply because $\frac{1}{2}mv^2$ is constant, as shown above, since $v^2$ varies inversely as $m$.

Textbooks of general physics, even those basic to technical subjects, do not seem to call specific attention to this important result of the analysis of simple harmonic motion. It is inherent, of course, in the general expression for the total energy at any given instant: $E_t=\text{P.E.}+\text{K.E.}=\frac{1}{2}kx^2+\frac{1}{2}k(A^2-x^2)$. For a given amplitude, the only way to change the energy in the system is to change the spring.

To say that in $E=4\pi^2mn^2A^2$, for the *same m* the energy is proportional to the square of the frequency, implies of necessity that different springs (different $k$-values) are used. With the same spring, $m$ cannot affect $E$.

Here a sharp distinction can be drawn between true simple harmonic motion and that of a simple pendulum. By taking the amplitude of the pendulum indefinitely small, we can make the character of the motion, at all points in the path of the pendulum bob, approach indefinitely close to simple harmonic. That is, we can approximate more and more closely the defining condition of simple harmonic motion; *viz.*, $a=-Cx$, where $C$ is a constant.[1] However, with the pendulum the vibrational energy, both kinetic and potential, varies as the first power of the mass of the bob. This fundamental difference as to energy, between the motion of the simple pendulum and that of a mass attached to a spring, which is true simple harmonic, always exists, no matter how closely the vibra-

tion of the pendulum approaches simple harmonic judged by $a \sim -Cx$.

In true simple harmonic motion, mass affects period; in the motion of the simple pendulum it does not—at any rate to the extent that we can regard the center of oscillation as being at a fixed distance from the point of suspension. The comparison may be tabulated as follows:

| Type of motion | Energy | Period |
|---|---|---|
| Simple harmonic | Independent of mass | Depends on mass |
| Simple pendulum | Depends on mass | Independent of mass |

This comparison assumes the same spring, that is, the same value of the force-constant $k$, and the same amplitude, for the different masses having simple harmonic motion.

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[1] The nearness of this approach is indicated by the following percentile departures of the pendulum's period from that for true simple harmonic motion [see A. G. Webster, *The Dynamics of Particles and of Rigid, Elastic and Fluid Bodies*, (B. G. Teubner, Leipzig, 1912), second edition, p. 48]: for angular amplitudes of 15, 5, and 1°, respectively, the departures are <0.50, <0.005, and <0.002 percent.

## A Method for Changing the Response of a System

RECENTLY there has been constructed at this laboratory an automatically-recording photometer for the measurement of scattered light. This instrument measures the intensity of light scattered by fogs whose droplets are essentially uniform in size. The diameter of the droplets is of the order of magnitude of the wavelength of visible light.

A characteristic of such scattering is the existence of very prominent lobes. A great range of intensities must hence be recorded, and the response of the suspension-type galvanometer which measures the output of the photocell was therefore made logarithmic. The following arrangement, whose general idea was suggested by Jupe,[1] was employed.

A curve was drawn on graph paper; the area under the curve varied exponentially with distance. To obtain an aperture having the shape of this area, but reduced in size, the area was cut out and the rest of the graph paper was photographed against a black background. The negative was placed directly in front of a condensing lens. Light from a source behind this lens passes through the exponentially shaped aperture, then through a second lens to the galvanometer mirror. There it is reflected to fall on the face of a photovoltaic cell of large surface area. The second lens forms an image of the aperture in the plane of the cell; as the galvanometer turns, the image moves across the face of the cell and generates a bucking voltage which is applied to the galvanometer. As a result, the galvanometer response becomes logarithmic. One can choose the amount of compression desired by drawing curves corresponding to different numbers of logarithmic cycles.

It is not the purpose of this note to describe the photometer, to be discussed in more detail elsewhere, but rather to point out how easily the aforementioned idea may be extended to serve other useful purposes. It is obvious that, by cutting out areas corresponding to various functions, any desired response may be obtained. One can imagine, for example, the possibility of recording on a linear scale the output of a device whose response is inherently nonlinear. Another application might be an interesting experimental study of low frequency nonlinear vibrations, using the galvanometer coil as the vibrating element.

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[1] J. H. Jupe, Electronics **12**, 44 (1939).

## A Departure in General Physics Laboratory Procedure

THE efficacy, the usefulness, and the pedagogical soundness of the usual laboratory scheme in general physics have long been doubted. With some daring I decided to depart from this familiar process and this note is intended to report on what I am doing. This scheme was pursued for a year in an altogether exploratory way but the evidence even for so short a time gives promise that the procedure can be profitable.

We did not start our laboratory sessions until the tenth week of school. The first nine weeks were given entirely to lecture, theory, and problems, and a discussion of the experiments which might be pursued in the investigation of this theory. The usual familiar laboratory setups were discussed with some thoroughness. The laboratory periods of these first nine weeks were used for problems and quizzes, and recitation. This obviously provided the student with a substantial backlog of physics, and precluded the very unsound practice of experiments coming far ahead of theory. This has long been the standard complaint of students and teachers, and a practically unavoidable situation.

Ten or a dozen experimental setups were then put up around the laboratory. Some lecturing was done on each setup, pointing up the measurements to be made, and the possible experimental difficulties. The students took rough notes.

Formal laboratory work began on the tenth week. It consisted of two two-hour sessions per week. Teams of two or three worked on each setup. No manual was used. The students were thus left to their own devices. The instructor ambled about the laboratory making suggestions here and there. We have assembled an array of laboratory manuals which students may consult, but experience shows that they are seldom used. Data are taken in no prescribed fashion. The write-up is brief, being sufficient for the student to crystallize his own thinking on the experiment and to state the results *discovered*.

*Comments:* 1. The student has considered the physics of the experiment and does not find himself in a totally alien situation. This has always been the dilemma—laboratory out of phase with theory. In addition, the student can now bring to the experiment a substantial array of knowledge from other places in the course. 2. There is no cookbook performance which so invariably degenerates to reading

the laboratory manual line by line and doing the experiment just that way! 3. The student is left to his own devices and thus he shows a spirit of inquiry and discovery. His performance possesses some of the spirit of research. 4. The laboratory is more enjoyable and not such a cold chore! 5. The student thus has about 18 two-hour laboratory sessions in a semester—that is, four hours of laboratory per week for half a semester. 6. Laboratory examinations, although not yet definitive, reveal a more general competence on the part of the student.

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## What Constitutes a Laboratory Examination?

THE mechanism of testing in general physics courses is pretty well established in all parts save the laboratory. We give theory quizzes, problem quizzes, standardized examinations, and the like, all of which answer quite satisfactorily the lecture and textbook portions of the course. On laboratory testing little appears to be done. Some teachers "throw" into the final examination "a few questions on the lab," but these invariably differ little if at all from the purely theory questions. Some laboratory examinations merely require the student to recite, with diagrams, what he did in a certain experiment.

We would be generally agreed, I am sure, as to what a laboratory examination *should* be. The laboratory is designed, obviously, to give the student training in a scientific method—that is, to examine a physical situation *experimentally* and to arrive at a conclusion by logical reasoning. This tells us, then, what a laboratory examination should do. It should provide a physical situation which the student can investigate experimentally and from the observation of which he can draw conclusions by logical analysis. Now this is more difficult to execute than to write about, as physics teachers generally will admit.

I wish in this note to report a scheme which I have found eminently satisfactory from both my own point of view and from the point of view of the student. It constitutes, as I see it, a fair test of laboratory ability and performance, and it turns out, happily, to be instructional for the student (which is what all examinations should be anyway!). For simplicity in writing and to minimize the length of this note I shall generalize on the scheme and give two illustrations.

*General Plan.*—Set out at the various stations in the laboratory a few pieces of equipment which by their very nature point up *something* to be measured. Do not supply all the necessary pieces. It is left as an exercise for the student to requisition the pieces he needs to pursue the investigation. Place at the station a brief typewritten statement of the problem to be pursued.

*Example 1.*—Set out two ammeters and two voltmeters of a range appropriate for the power source available. Instructions: Find experimentally how the readings of two ammeters will compare if they are placed in series in a circuit. Is the situation altered if the meters have different resistances? Repeat the investigation for two voltmeters in series.

*Example 2.*—Set out a harmonic motion coil spring and a support, together with a stop watch or timer, a known weight and an unknown weight. Instructions: Determine the weight of the unknown.

The implications in these two illustrations, taken at random, are clear to all physics teachers. It is apparent too, that these arrangements have endless possibilities. The scheme is workable with small groups. With judicious choice of setups a student can run through two or three experiments an hour, six perhaps, in a two-hour laboratory examination period. The report can be oral or briefly written, as the instructor wishes.

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## Replies to Inquiring Letters

THE interesting letters on p. 534 of the November issue call for a reply. Mr. Miller's inquiry about the motions of sand and lycopodium on a Chladni vibrating plate was fully explained and tested by Michael Faraday.[1] The motions were also discussed by John Tyndall[2] in his book *Sound*. His query about the precipitate in the rotating contents of a beaker is harder to satisfy. Dr. Saul Dushman gave me an explanation many years ago but, I cannot reproduce it. I think it brought in Bernoulli's principle and vertical radial vortices.

The query about the squeaking of dry powdery sand may be answered by referring him to "Musical Sands" in Poynting and Thomson's *Sound*.[3] These sounds are often heard in Toronto as cartwheels move over the cold hard snow. I imagine the crystals may vibrate under the stresses brought to bear on them.

With regard to the answer required by Robert Katz for his heat experiment I would guess that the match on the asbestos would have caught fire first. The heat from the burner would be pretty well localized in the center of the asbestos plate which would therefore get hot. The aluminum plate would conduct the heat away from the center and so remain, for a time, too cold to ignite a match.

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1 M. Faraday, Trans. Roy. Soc. (London) 121 (1831).
2 John Tyndall, *Sound* (Longmans-Green, and Company, London, 1898), seventh edition, p. 149.
3 Poynting and Thomson, *Sound* (Griffin and Company, London).

# ANNOUNCEMENTS AND NEWS

## Book Reviews

**The Climate Near the Ground.** Rudolf Geiger. Pp. 481. Harvard University Press, Cambridge, Massachusetts, 1950. Price $5.00.

*The Climate Near the Ground*, by Rudolf Geiger, Professor of Meteorology and Director of the Meteorological Institute, University of Munich, is a translation of the amplified second German edition which was prepared after the Second World War, and published in English in October, 1950. The first edition was published in 1927. Several people have participated in the translation and they have done a marvelous job of translating, not only in the text but also in the legends and diagrams. The book can be considered as both a text and a reference book.

As the title indicates, this book deals with the climate near the ground to a height of two meters from the ground surface. Two meters is supposed to be the upper limit; however, from a different point of view and in many miscellaneous instances such as valleys, mountain slopes, etc., the upper limit may exceed two meters.

The book presents and discusses the fundamental factors governing the climate near the ground and the great differences that exist in this climate, even within short distances, as brought about by variations in the land surfaces. For example, the climate over a muck soil, a sandy soil, a bare soil, a soil covered with different kinds of vegetation of various heights, a soil that receives different degrees of shading, sunniness, or wind protection, etc., will differ greatly. This is a study of climate "in the least space" in contrast to the large scale climate which the meteorologists deal with. The book discusses with thoroughness the factors of heat exchange, heat conductivity, radiation and irradiation, heating and cooling process, temperature inversion, soil and air temperature, reflection, utilization of incident heat radiation by different kinds of ground, humidity and wind relationships, and a great many other factors and the role these play on the climate near the ground under varied circumstances.

The book is very readable. Its style is informal, simple, and exceedingly clear. It is noncontroversial. The arrangement of the contents is very good and logical. It is scientific. The material is comprehensive, thorough, accurate, and up-to-date. The author knows his subject well, having been himself a researcher and contributor to the development of the subject. Although the book is about average size, it is packed with an enormous amount of valuable information, having covered almost every conceivable factor that plays any role on the climate near the ground. The main points are supported by citing experimental data drawn from various sources in different parts of the world. More than 821 references are cited. In addition to this large number of references, the book contains 181 diagrams. These are all excellent illustrations and aid greatly in clarifying the various fundamental points discussed.

The book would be of special interest and value not only to students of meteorology, but also to botanists, ecologists, foresters, agronomists, soil scientists, horticulturists, zoologists, and entomologists. This is because the climate near the ground has such a direct bearing in the respective fields of all these groups. The book should also be of great interest to students of physics. Here is a wonderful example of the application of the laws of physics to applied fields.

The book explains the heaving of soils as due to the expansion of water upon freezing. According to the studies of the reviewer this is not the whole explanation. The true explanation appears to be that heaving is caused almost entirely by the drawing or accumulation of water on freezing at or near the surface by the force of crystallization. This frozen water grows upwards in the form of massive capillary ice columns, pillars, ridges, or solid sheets of ice. As water is pulled or drawn to the points of freezing, and as these different forms of ice grow upward, they push upward. This type of heaving exerts a powerful leverage and pulls the plants out of the ground. The book stresses that soil temperature exerts a strong influence on frost prevention; however, this is true mainly in mild frosts and not in severe frosts. This differentiation has not received the proper emphasis. These criticisms, however, are insignificant, and do not subtract from the great value of the book.

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**Photons and Electrons.** K. H. Spring. Pp. 108. John Wiley & Sons, Inc., New York, 1950. Price $1.75.

This book contains a concise account of the interaction of electrons with photons at high energies. The topics discussed include the photoelectric effect, the Compton effect, the production of bremsstrahlung and x-rays, pair production, and the formation of cosmic-ray showers. The author has attempted to make accessible to readers with a limited theoretical background some of the advanced and rather complicated results contained in such books as Heitler's *Quantum Theory of Radiation*, and in the article on cosmic-ray theory by Rossi and Greisen in the *Reviews of Modern Physics* (1941). Important equations from these works are briefly presented, without a derivation, but with a general explanation of their meaning and importance as well as of some of the reasoning leading up to them. Moreover, the author presents considerable experimental material (with references to the original literature) to indicate to what extent these results have been verified. The book thus serves a valuable purpose in providing a convenient compilation of theories as well as facts, and it is written in a lucid and readable style. It should be read with some caution, however, because the text is marred by a number of slipshod and incorrect statements, especially with regard to the underlying electrodynamic and quantum-theoretical ideas.

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**Thermodynamics.** FRANCIS WESTON SEARS. Pp. 348. Addison-Wesley Press, Inc. Cambridge, Mass, 1950. Price $6.00.

Designed for the upper division students in Electrical Engineering at the Massachusetts Institute of Technology, this text covers not only the classical reaches of thermodynamics, but also the elementary parts of kinetic theory and statistical mechanics, extending through basic quantum statistics and some illustrations in the domain of low temperatures. The mathematical expressions which are used are carefully developed from a basis of calculus, and the diagrams and illustrations which are used profusely to clarify the text material are carefully prepared and easy to follow. The work is characterized by the skill in organization and presentation which the users of other Sears textbooks have come to expect. The mechanical design of the book is good. It is well printed and easy to read.

From the standpoint of a physicist, the fault of the book is that the presentation is pehaps too well finished. The reader is often left with the impression that people who pronounce thermodynamics without the last three syllables are entirely correct. The science is complete and has no dynamic possibilities for future growth. This attitude is probably justified when the book is used for its avowed purpose of pushing a little fundamental science into a very crowded technological curriculum, but the emphasis should be changed somewhat if the book is used to teach students in a science curriculum, especially if the students are planning graduate study. This can be done if the book is accompanied by lectures and collateral material in which the provisional and tentative nature of all scientific theory is clearly pointed out and the possibilities for future growth of thermodynamics through the use of such devices as the open system and the theory of communication are discussed. If this relatively minor problem can be overcome by the instructor, the book should offer a valuable basis for an upper division course in thermodynamics and kinetic theory for science students as well as for engineers.

The problems at the end of each chapter are numerous and well chosen, both from the standpoint of the practical application of the material and the thinking required by the student.

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**Electricity and Magnetism.** NORMAN E. GILBERT. Third Edition. Pp. 569+xv. Figs. 392. The Macmillan Company, New York. Price $5.00.

It is unusual to discover a new edition of an old book which has eighteen pages fewer than its preceding edition.[1] This is achieved by closer spacing of type and equations; actually, the new edition contains an additional chapter and some new material. The new chapter is created by breaking of the former discussion on conduction in gases, electrons, photons, and positive ions into two chapters. In general, the diagrams, discussion, and problems are identical with those in the last revised edition (1941) although new data have been substituted in the problems and answers are now given only for alternate problems. The calculus is assumed as a corequisite or prerequisite and the book accomplishes its purpose to "give to the student who plans to continue the study, either in the direction of engineering or in the direction of theory and investigation, a firm foundation upon which to build." The old concept of magnetic pole is introduced in the second chapter and the force on a unit pole is employed later to define unit current. The chapter on units and dimensions has been moved from the middle to the end of the book. The mks system of units, mentioned in this last chapter, is not used in the problems, although understanding gained by its use in theory and in problems might be helpful to the student planning to enter the electrical engineering profession.

One of the few omissions in this new edition is the discussion of the Zeeman effect, which may well be omitted from a book of this scope. Additions include discussions on the betatron and synchroton, Bethe's theory of the sun's heat, chain reaction, plutonium, and the pile. There are no problems on this material although the discussion is adequate for this type of textbook. Complex numbers are introduced in this third edition for the solving of alternating current networks. While the preface states that the question of what to include and what to omit is an omnipresent one, the reviewer would mention a few things which inquisitive students or some instructors will find missing. In the ample chapters on communication and amplification no mention is made of the transistor. No discussion of the broadcast of sound in a television program accompanies the discussion of television and amplitude modulation. The topic of frequency modulation is not discussed. Common words such as *radar* and *deuteron* are missing and only a brief mention is made on page two of the mesotron (this longer name is used). One could debate as to whether or not the discussion of the packing fraction curve should be replaced by a discussion of the variation of binding energy per nucleon with mass number. In the chapter on the thermoelectric effect Fig. 29.5 is similar to those which the reviewer[2] has found in many texts. While the discussion of this parabolic curve for a copper-iron couple is correct, the objections are obvious for its industrial or practical application. Thus, in our present age of visual learning the student tends to retain the picture of the exceptional rather than the practical, since a typical practical thermoelectric curve is not reproduced.

The student will find the text easily readable and the ideas are amply illustrated by good problems at the ends of most of the chapters. These problems involve thought and understanding and not mere arithmetic. Those experiments for which laboratory apparatus is ordinarily available will fit well into the text discussion. The book will not satisfy the instructor who wishes to begin the study of magnetism with the study of the forces between moving charges and who prefers to discuss magnetic poles in connection with magnetic properties of matter in general.

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[1] Review in Am. J. Phys. 10, 167 (1942).
[2] Schoepfle, Am. J. Phys. 16, 121 (1948).

**Introduction to Electricity and Optics.** NATHANIEL H. FRANK. Second Edition. Pp. 440, 6×9 in. McGraw-Hill Book Company, Inc., New York, 1950. Price $5.00.

In the preface to the first edition of this book, the author stated: "This book has been written primarily as a textbook for the use of those second-year students at the Massachusetts Institute of Technology who intend to pursue further studies in electrical engineering, physics, or both. These students have completed a year's course in calculus and one in mechanics and heat utilizing the author's text[1] and are simultaneously pursuing a second course in calculus." It would be presumptuous for an outsider to review the book with respect to its stated use. Furthermore, the number of institutions which have a two-year program in introductory physics comparable with that of the Massachusetts Institute of Technology is somewhat limited. However, the book should enjoy a use beyond that of the second year of such a two-year physics course. It could be the basis for a year's course in electricity and optics for students who have previously taken the more standard one-year course in general physics, or it could be the basis for a semester's course in electricity and magnetism, again for students who have had the standard course. Used in this latter way it would be preferable to many of the so-called intermediate texts in electricity and magnetism, both with respect to subject matter and method of presentation of that subject matter, and to the demands on the student in the form of problems involving more than substitution of numbers into equations developed in the text. The problem list, incidentally, is phenomenal.

As might be expected, many theorems or equations are quoted without proof, being suggested either by the consideration of simple cases or by plausibility arguments. Examples of the former are the concepts of spatial distribution of electric and magnetic energy and Poynting's theorem; of the latter, the radiation field of an oscillating dipole. There can be no quarrel with this procedure unless one is considering the book as an advanced undergraduate text as opposed to an intermediate one. In fact, with additions to or amplifications of the author's discussions of the development of such equations on the part of the instructor, this book could serve as an advanced undergraduate text in electricity and magnetism.

Rationalized mks units are used throughout. It is too late to argue the desirability of either mks units or rationalized units, but it is refreshing to find a book using these units without indulging in some nonsense about their being more fundamental than the older units. The treatments of **E**, **D**, **B**, and **H** are excellent.

Geometrical optics is treated very briefly. Here, for once, the author has gotten into trouble in quoting results more general than he has proved. In referring to an optical system in which the indices of refraction of the object space and the image space differ, a case he has not considered mathematically, he confuses the principal points and the nodal points. Photometry is treated in three pages at the end of the chapter on heat radiation. No consideration is made of the photometry of optical systems, and it is in this connection that inexperienced physicists attempt to make impossible demands of an optical system.

If one be permitted to be slightly facetious, the chief objection to this book is the statement in the preface, quoted above. An undergraduate in a small college might object to using in a junior course a book whose preface indicates that it is designed for the second half of an introductory physics course.

H. A. NYE
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[1] N. H. Frank, *Introduction to Mechanics and Heat* (McGraw-Hill Book Company, Inc., New York, 1939), second edition.

**Heat and Temperature Measurement.** ROBERT L. WEBER. Pp. 422+x. Prentice-Hall, Inc., New York, 1950. Price $5.00.

The author has made an excellent choice of material for a combined textbook and laboratory manual to be used in a lecture-recitation and laboratory course which follows an introductory physics course. The many demonstration problems within the chapters and the long lists of reference questions and problems with answers at the end of the chapters are evidence of the excellent teaching background of the author.

The book has been written with emphasis on the experimental methods rather than on theory. However, the study of the chapters on thermometry, thermoelectricity, radiation, elementary thermodynamics, thermal analysis, and temperature control, combined with the twenty-nine experiments of Part II, should give the student an adequate theoretical as well as a practical knowledge of heat and temperature measurement, including the extremes.

The titles of the experiments combined with the details of the apparatus and the well-written directions make the book attractive as a working manual for a heat laboratory course. The many correction factors associated with heat and temperature measurement and control are adequately discussed from a theoretical point of view and the usual experimental methods of measuring them are described in detail. This, combined with the many tables of thermal data of the common materials used, makes the book a handy reference for anyone attacking a new experimental problem in heat and temperature measurement or control.

The section in the appendix on errors would be more desirable if it included the distinction between precision and accuracy, examples of systematic and accidental errors, and the interpretation of errors. The subject, "Experimental Errors," is one of the most important in any laboratory course and a thorough treatment in a book of this type would decrease the burden of the laboratory instructor considerably.

The reviewer objects to the author's discussion of the initial topics in Chapter I, "Heat" and "Temperature." When an author states that heat is energy transferred by a thermal process and follows with Maxwell's definition of temperature, which is: "The temperature of a body is its thermal state considered with reference to its ability to communicate heat to other bodies," he has only added

confusion to the student's concepts of heat and temperature, because each is defined in terms of the other. A more logical development of these concepts would begin by making use of the student's understanding of the meaning of the word, "warm," which he uses to describe a sensation. The qualitative aspect of the concept of temperature may then be expressed by the definition, "Temperature is that physical property of matter which permits one to distinguish bodies by warmth sensations." The quantitative aspect of the concept of temperature may be expressed by the definition of a temperature scale in terms of fixed points and the thermal property of a thermometric substance, e.g., the gas temperature scale. Note that the word, "heat," has not been used to develop this concept of temperature. To help the student develop the correct qualitative aspect of the concept of heat, it now may be defined in terms of temperature as follows: "Heat is energy in transit between a system and its surroundings by virtue of a temperature difference." The correct quantitative aspect of the concept of heat may be expressed by the law of conservation of energy applied to any isothermal process. With this approach an instructor can answer the alert student's question, "When is energy called heat?" Furthermore, it would eliminate the confusion caused by the statement that heat has been identified with the internal molecular energy of the body and the statement of the first law of thermodynamics which introduces internal energy as something different from heat. This adverse criticism is not made to detract from a book otherwise well-done but to help prevent the propagation of these illogical definitions from one textbook to another, which is not unusual, particularly in the field of elementary physics.

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# RECENT MEETINGS

## Southern California Section

The annual fall meeting of the Southern California Section of the American Association of Physics Teachers was held in the George Pepperdine College Auditorium in Los Angeles on Saturday, October 21, 1950. It was attended by approximately ninety members and friends. The morning program consisted of an invited paper, **Dangerous Currents in the Surf** by DR. FRANCIS P. SHEPARD, *Scripps Institute of Oceanography of the University of California*, and the following ten-minute contributed papers showing demonstrations for elementary physics classes.

**Water Runs Up Hill, or Does It?** JOHN G. BETTS, *John C. Fremont High School, Los Angeles*.—Using five-gallon glass bottles a demonstration of Hero's fountain is presented which will run for two to three hours without attention.

**Projection of Small Scale Phenomena.** HOWARD S. SEIFERT, *Jet Propulsion Laboratory, California Institute of Technology*.—A microprojection device employing a small carbon arc and a 12-mm movie camera lens was used to project a 3-mm slide area on a 1.5-m screen (500 diameters). Small biological and crystal structures were clearly shown, and living one-celled and higher animals were shown in action.

**Education?—Or Merely Training?! VIII, A Demonstration that Resolved a Dilemma.** GEORGE FORSTER, *Pasadena City College*.—The demonstration was devised many years ago when the author was hard pressed by a group of students who persistently asserted that a certain problem in magnetism presented an impossible situation because it violated the conservation of energy principle. The problem follows: Two permanent bar magnets each 30 cm long and with concentrated end poles of 350 cgsup strength are suspended from the knife edges at the ends of a beam balance. One of the magnets hangs vertically above the left-hand pan with its north pole up. The other magnet is supported horizontally above the right-hand pan with its north pole directed toward the center of the first magnet. Thus the axes of the magnets lie in the same vertical plane with their centers at the same level and 30 cm apart. Assuming that the two magnets balance exactly before they were magnetized, what mass must be added to one pan to maintain the balance when they are magnetized, and in which pan must the mass be placed?

The demonstration reproduces the conditions of the problem in all respects excepting the magnitudes of the pole strengths and the distances involved. It also strikingly illustrates Newton's third law of motion.

**Demonstration of a Rocket-Propelled Airplane.** VERNON L. BOLLMAN, *Occidental College*.—A small plastic toy airplane is suspended by means of two hooks on a piece of braided copper wire stretched horizontally across the front of the lecture room. The airplane is free to slide along the wire. A small block of wood is glued and bolted to the bottom of the plane and is drilled with a three-quarter inch hole extending into the block about two and one-half inches from the rear with the axis parallel to the supporting wire. A small carbon dioxide cylinder is inserted in the hole. These 'chargers' are available in most drugstores. The rocket is fired by puncturing the cylinder with a large sharpened nail given a smart blow with a small hammer. A piece of cloth about a yard square is draped over the wire on the far end to stop the plane.

**A van de Graaff Generator for Demonstration Purposes.** RICHARD H. WATERS, student at the *California Institute of Technology*, introduced by FOSTER STRONG.

**Demonstration of the Hydrostatic Paradox: Phase II.** LAURENCE E. DODD, *University of California at Los Angeles*.—Customarily the hydrostatic paradox is demonstrated with two or more connected columns of water having the same depth but widely different cross-sectional areas. The different weights of the columns seem to balance each other. This is because the observer's mind reads into the situation some such analogy as that of the beam balance.

The experiment is usually discussed from the standpoint of force in the vertical direction, that of gravity, in keeping with a comparison of weights. But the same laws of pressure distribution produce an equally striking paradox in the horizontal direction. These paradoxes are two aspects, or phases, of the same hydrostatic situation. Both phases depend on the relation between pressure and depth, and both involve a comparison of weights. The first may be called Phase I, and the second, Phase II. Phase II is involved when it is pointed out that the total horizontal force of the water against a dam depends only on its total depth at the dam, and is entirely independent of the volume of water impounded. Thus the same horizontal force would be exerted on Hoover Dam if the reservoir extended only 125 ft, say, back of the dam, instead of the 125 miles of Lake Mead. In the present demonstration, Phase II of the paradox is shown directly by projecting a transparent cell containing a vertical, elastic membrane separating the cell into two compartments, one of them much narrower than the other. Water added to the larger causes a bowing outward, in a manner consistent with the known variation of pressure with depth, whereupon adding water to the smaller compartment until the two surface levels are the same, causes the membrane to become straight again. Thus a smaller volume of water is able to "hold back" a much larger one.

At the conclusion of the scheduled portion of the morning session DR. HOWARD SEIFERT was asked to comment upon the development of rockets to the moon. Dr. Seifert's reply was to the effect that each step farther away: five miles, ten miles, fifty miles, two hundred miles, and so on, requires a tremendous amount of effort on the part of a great many people, but that it seemed possible that with adequate financial support development could be continued until the moon trip was feasible.

In addition, several high school teachers present suggested that physics and science in high schools might well be encouraged if colleges in the neighborhood would undertake to bring to high school student bodies good, well-presented demonstration lectures on such subjects as liquid air. While no college members present were able to accept this responsibility at the time, the Secretary is glad to learn that at least one college has done so.

Finally, at the morning session DR. WILLARD GEER was asked to comment upon the situation in color television as a result of the recent ruling of the Federal Communications Commission. Dr. Geer, who was one of the parties to the Commission's hearings on color television at Washington, D. C., replied that the ruling had not been very favorably received by the industry and that matters were too undecided at the moment to be able to guess what the outcome might be.

The afternoon program consisted in the movie **The Story of Palomar** which was followed by a talk by DR. IRA S. BOWEN, *Director of the Mt. Wilson and Palomar Observatories.* Dr. Bowen described the final testing and polishing of the mirror on Mt. Palomar, showed a number of slides of photographs taken with the new telescope, and commented on the research program to be undertaken.

A business meeting of the Section and an executive committee meeting concluded the activities of the day.

DAVID F. BENDER, *Secretary*

## Illinois Section

The Illinois Section of the American Association of Physics Teachers met for their annual fall meeting at Bradley University, Peoria, Illinois, on October 27 and 28, 1950. Thirty physics teachers of the state were in attendance.

The Friday evening session featured a demonstration lecture on "Magnetic Recordings" by MR. R. J. TINKHAM of the *Magnecord Corporation*, Chicago. On the Saturday program, there were eight papers covering experimental work and teaching devices. There was also a panel discussion on the subject "Preparation of High School Physics Teachers and Certificating by the State." The local Chairman presiding at the meeting was PROFESSOR C. N. PATTERSON of *Bradley University.*

The Illinois Section decided to organize along the lines of the national organization with an executive committee to give continuity to the programs from year to year. With this in mind, an executive committee of the representatives of the state was selected, as follows: W. H. ELLER, *Western Illinois State College;* W. J. HOOPER, *Principia College;* GLENN Q. LEFLER, *Eastern Illinois State College;* O. B. YOUNG, *Southern Illinois University;* R. J. MILLER, *Greenville College;* C. N. PATTERSON, *Bradley University;* LYLE FINLEY, *Monmouth College*, Secretary-Treasurer; O. L. RAILSBACK, *University of Illinois*, Chairman.

Two committees were appointed:

1. O. B. YOUNG, Chairman. To prepare a report on the preparation of high school physics teachers of Illinois, with plans for getting the recommendation adopted by the State Certificating Board.

2. GLENN Q. LEFLER, Chairman. To prepare a constitution and by-laws for the state organization, consistent with the new organization plan centered in an Executive Committee which will be responsible for continuity of programs.

O. L. RAILSBACK, *Chairman*

## Indiana Section

The Indiana Section of the AAPT met at Valparaiso University and Valparaiso Technical Institute on May 6, 1950.

The group toured the Technical Institute during the morning, lunched together in the Institute Cafeteria and inspected the Physics Department of the University in the early afternoon.

The formal part of the program consisted of the following papers.

**A Special Case of Self-Inductance. A Gadget or Two.** O. H. SMITH, *DePauw University.*

**Report on Requirements for Physics Teachers.** M. E. HUFFORD, *Indiana University.*

**Some Uses of Surplus Equipment.** A. D. HUMMELL, *Ball State Teachers College.*

**Graduate Training for College Physics Teachers.** J. F. MACKELL, *Indiana State Teachers College.*

**Wave Shaping Demonstration.** H. FULTON, *Valparaiso Technical Institute.*

**Laboratory in Elementary Physics.** I. WALERSTEIN, *Purdue University.*

**Television—Demonstration.** J. W. ALINSKY, *Valparaiso Technical Institute.*

**Some Novel Uses of a Ballistic Galvanometer.** CLAYTON M. ZIEMAN, *Wabash College.*

**Humphrey Davy's Heat-by-Friction Experiment.** DUANE ROLLER, *Wabash College.*

**Demonstrations on Resonant Pendulums.** ANCIL THOMAS, *Valparaiso University.*

**Teacher Training for the Technical Institute.** J. B. HERSHMAN, *Valparaiso Technical Institute.*

Speakers at the evening banquet were DR. O. H. SMITH, recent recipient of the Oersted Medal, who reported on the winter meeting of the AAPT, and DR. DUANE ROLLER, whose topic was "Should Physics be Assigned a Fundamental Role in the Liberal Arts Curriculum."

It is likely that the section meeting in the spring of 1951 will be held at Franklin College.

R. W. LEFLER, *Secretary*